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ECODIGITAL FUTURES

MACROECONOMIC MODELLING OF R&D FOR THE TWIN TRANSITION

# D4.1 | Conceptual modelling of the Twin transition


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**D4.1 | Conceptual modelling of the Twin transition:**

# An evolutionary stock-flow consistent input-output supermultiplier model of the global economy

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**Abstract / Executive Summary**

This document presents a conceptual macroeconomic model that is to be developed into a full-scale policy simulation model for the impact of green and digital technological change on the global economy, particularly Europe. The main focus is on the basic workings and structure of the model, and the way that the model looks at macroeconomic dynamics. Green and digital technological change are not addressed in this version of the model, i.e., this is left to future development. The conceptual model is essentially supply-based, i.e., the future incorporation of technological change will drive economic growth and change as portrayed by the model. But there is also an important role for demand, because the main way in which the model see coordination of the economic process is by a multiplier. In particular, we adopt the notion of a supermultiplier, where in addition to consumption demand, also investment demand is a part of the multiplier. The model is also stock-flow consistent, which means that it keeps detailed accounts of financial stocks and flows, and that financial debt and wealth stocks are determinants of economic behaviour. The stock-flow consistent nature of the model allows us to numerically calculate a steady state, which we propose to be the main vehicle for future model development.

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## 1. Introduction

In this deliverable of the TWINRD project, we will describe a conceptual macroeconomic model that can be elaborated into a full-scale policy simulation model of the twin transition, a term coined by the European Commission (e.g., Muench et al., 2022) to describe the growing impact of green and digital technologies on today's global economy and society at large. The model as described in this document is conceptual, and it does not include any modelling of the twin transition yet. Instead, the focus is on the development of a new type of macroeconomic simulation model, and the aim is to provide a proof-of-concept of the ideas underlying this model. In a next stage of the TWINRD project, the conceptual model can be elaborated into a full-scale model aimed at analysing the twin transition.

The model combines several ideas and major concepts from the existing literature in an original way. We bring together the ideas of the so-called supermultiplier (Freitas and Serrano, 2015), stock-flow consistency of the macroeconomic accounts (Godley and Lavoie, 2007), dynamic input-output modelling (Duchin and Szyld, 1985), and the role of structural change (Cantore and Alcorta, 2021). In the resulting model, activity levels in the global economy are determined by the multiplication of autonomous demand and a multiplier matrix, which provides a Keynesian flavour. However, by the inclusion of ideas from the stock-flow consistent (SFC) literature, autonomous demand is endogenized, so that ultimately, the model portrays the global economy as supply-led.

Although we do not fully endogenize the long run yet (e.g., productivity growth is largely absent from our analysis), the supply-led nature of the economy opens doors to model technological change and the twin transition in the future, full-scale version of the model. In this paper, we limit ourselves to analysing if and how the model can reach a steady state. This is analysed by simulation analysis of a small-scale fully theoretical version of the model, and by the calculation of the steady state in a numerical procedure that is based on the principles of SFC modelling. The numerical estimation of the steady state is performed for the small-scale theoretical model as well as for a larger-scale empirical version.

The rest of this paper is organized as follows. In the next section, we briefly survey some literature on which our model is based. This section also explains in general terms how our model builds on the existing literature, and how it differs. In section 3, we present the accounting framework on which the model is built, which is a so-called multi-regional input-output (MRIO) table. Here we explain how this part of the accounting logic works, and how we construct the empirical MRIO that will be used in the analysis. Section 4 explains the model in formal terms. This starts by an explanation of the way in which the short-run activity levels (output and employment) in the global economy are determined and then proceeds to explain how these short-run states are linked by inter-temporal updating of the dynamic parts of the model. Although the dynamics are not fully specified in this conceptual version, we also explain how future development should enhance the intertemporal dynamics.

In section 5, we present some simulation experiments for the small-scale theoretical example, a context with two countries and three sectors. In these experiments, we see a steady state emerging, and hence the logical follow-up in section 6 is to explore this steady state and investigate how it can be found numerically. This includes theoretical development of the steady state and the numerical procedure to find. We apply this to the theoretical $2 \times 3$ setting, as well as to a larger "real-world" empirical implementation of the model. In the real-world application, we also introduce, admittedly in an ad hoc way, a role of the government, which is absent in the small-scale theoretical model.

The final section 7 summarizes the conceptual model and presents a proposal for a list of actions in the next development stage.

## 2. Related literature

### 2.1. Situating our approach

Large policy simulation models come in many different guises. Ciarli and Savona (2019) survey a large subset of these models, focusing on those that explicitly address climate change and on the role that structural change plays in these models. While the conceptual model that is described in this document does not yet address climate change, it does reserve a large role for structural change. Also, in the subsequent development stage of the model, green technological change will play a large role. Thus, the survey by Ciarli and Savona seems to sketch an appropriate context for positioning our conceptual model in the related literature.

The five model categories that Ciarli and Savona distinguish are the Integrated Assessment Models (IAM), Computable General Equilibrium (CGE) models, Structural Change Models (SCM), Ecological Macroeconomic models in the Keynesian tradition (EMK) and Evolutionary Agent-Based Models (EABM). In terms of our conceptual model, the first two of these categories, IAM (e.g., Nordhaus, 1979; as a pioneering contribution) and CGE (e.g., the detailed survey in Boulanger and Bréchet, 2005), are least relevant because these models are generally based on equilibrium by maximization strategies employed by economic agents. Such equilibrium thinking is not applied in our conceptual model.

On the other hand, the EMK, SCM and EABM traditions all provide inputs for our modelling approach. Following the survey by Hardt and O'Neill (2017), Ciarli and Savona characterize the EMK approach by its emphasis on monetary factors in addition to the "real" macroeconomic sphere; application of Godley and Lavoie's (2007) Stock-Flow Consistent (SFC) framework; the absence of full employment and other equilibrium assumptions; and an emphasis on environmental and/or financial instability. Of these elements, especially the SFC framework is important to our conceptual model, as we fully embrace those ideas and apply them as a crucial part of our model closure. In addition, we use the concept of the supermultiplier, which is also a Keynesian idea, although it does not figure yet in the climate change models in the survey by Ciarli and Savona.

While the "structuralist approach" (e.g., Cantore and Alcorta, 2021) is characterized by a coherent theoretical approach to the phenomenon of structural change, the SCM group of models as Ciarli and Savona describe them is a mixture of theoretical modelling approaches that only share an emphasis on the changing sectoral composition of the economy. Within this broad category of SCM Ciarli and Savona place such widely varying theoretical approaches as dynamic input-output modelling (e.g., Pan, 2006), (semi-)endogenous growth models (Bretschger and Smulders, 2012), and ecological models (Ayres and van den Bergh, 2005). Our picking from this SCM category of models is the dynamic input-output model, which is a core of our approach.

Ciarli and Savona present the EABM category of models as a way to address the dynamically changing nature of the economy and its interaction with the ecological sphere. These models are rooted in evolutionary economics (Nelson and Winter, 1982), which is an approach that allows for the main economic relations, i.e., the things that models try to describe, to change over time under the influence of (co-)evolutionary processes, which are modelled by interactions between agents. This kind of evolutionary change is not something that our model is able to capture, but

our approach does include some elements of evolutionary economics, in particular the role of innovation and technological change as a supply-side factor that drives economic growth.

In conclusion, the conceptual model that will be presented below builds on and combines elements of different approaches for macroeconomic policy modelling. We now proceed to discuss two specific issues that play a large role in our model: the SFC modelling framework and the idea of the supermultiplier.

### 2.2. Stock-Flow Consistency (SFC)

The SFC notion (Godley and Lavoie, 2007) plays a large role in modern Keynesian thinking. The basic idea is that current (short run) economic transactions lead to financial flows between groups of agents (households, firms, the government, and banks including central banks), that these flows accumulate into stocks of debt and wealth, and that these stocks are an important determinant of economic behaviour. As an example that will be particularly relevant in our conceptual model, economic transactions in a given period will include income (wages, interest, dividends) flowing towards households, and expenditures (consumption) made by households, resulting net savings that may either be negative or positive. These net savings accumulate into the stock of financial wealth of households (savings accounts at banks, financial investment portfolios), with negative (positive) savings diminishing (increasing) those stocks. These financial stocks may then influence consumption in the future, e.g., wealthy households may spend more on consumption.

Because of the central role of financial flows and stocks, the SFC literature is strongly interested in finance and monetary economics (Godley and Lavoie, 2007). In terms of finance, the interest is in decisions regarding financial investment portfolios (e.g., investment in equity vs. bonds and government bonds vs. private bonds). In terms of monetary economics, the interest is in how monetary and fiscal policy influences those portfolio decisions. This implies that SFC models are often very detailed in terms of the modelling of financial and monetary variables and processes.

This tendency is clearly visible in applied policy simulation models in the SFC tradition (see e.g., Mazier and Tiou-Tagba Aliti, 2012 for a global model; Byrialsen and Raza, 2020, 2022 for a model for Denmark; Zezza and Zezza, 2022 for a model on Italy; Di Domenico et al., 2024 for a model calibrated loosely to the US; Nikiforos and Zezza, 2018 for a survey). These models usually consist of a detailed set of equations describing the financial and monetary sector, as well as behavioural equations for production and consumption decisions. The equations form a simultaneous system which is usually solved numerically. With proper estimation of parameters using econometric methods, simulated predictions from this system of equations will mimic the empirical data. Using policy scenarios, e.g., in terms of specific values for exogenous variables, such a model can be used for policy simulation.

It is important to note that in the approach that these applied SFC models take, there are no specific assumptions about equilibrium or steady states involved in the model simulations, as is often the case in other modelling traditions, such as Ciarli and Savona's IAM and CGE traditions. The latter approaches can be seen as modelling an equilibrium or steady state of the economy that will generally not be reached but can be seen as a target to which the economy tends in the medium-to long run. Simulating such models is thus supposed to provide insights in the medium-to long run developments of an economy. The applied SFC modelling tradition, on the other hand, does not apply any such idea of a distinction between a steady state equilibrium and the actual evolution of the economy.

In the application of the SFC idea in our own conceptual model, we will diverge in two ways from the main developments in these applied SFC models. First, we do not include a detailed modelling of the financial or monetary side of the economy. Instead, we only use the simplest representation of financial stocks and flows. For example, we reserve no role of banks including central banks, we do not include any interest payments, and do not consider distinctive portfolio options. Second, we will provide one option for model analysis that relies on the notion of a steady state. As we will show, our conceptual model can be simulated in terms of a simultaneous system of (difference) equations, but we can also numerically calculate a steady state. Simulations show that the model indeed tends towards this steady state, which is defined and calculated using the main principles of stock-flow consistency. However, whereas the notion of a steady state is largely absent from the applied SFC literature, we embrace it as a way to analyze the main economic trends that the model predicts. This is a feature of our model that borrows from the CGE tradition.

### 2.3. The supermultiplier

Another feature of our model that we borrow from the Keynesian literature is the supermultiplier. There are two aspects to this: the way in which we use a multiplier, and that fact that our multiplier is a supermultiplier. The multiplier as we us it is the main mechanism of short-run economic coordination, i.e., a way of closing the model in the short run. This follows Robinson (1937) as discussed in Meijers et al. (2019): instead of markets being cleared by price adjustments, activity levels (output and employment) are determined by a multiplier and by autonomous demand.

In our multi-sectoral conceptual model, the multiplier is a matrix and follows the logic of input-output economics. The application of the supermultiplier in a context of global input-output models has been proposed in a series of recent contributions (e.g., Dejuán at al., 2022; Portella-Carbó, 2016; Portella-Carbó and Dejuán, 2019; Mariolis and Ntemiroglou, 2023) and is also the approach that we will follow. In the common (so-called open) input-output approach (Miller and Blair, 2009), multiplier effects only consist of intermediate input linkages, and hence in terms of value added or GDP, there is no real multiplier (one unit of final demand leads to one unit of value added). In our model, which is a closed input-output model (Miller and Blair, 2009), a part of consumption demand is endogenous, which increases the value of multiplier. In this case, one unit of autonomous final demand will increase value added or GDP by more than one unit.

The notion of autonomous demand needs to be defined precisely here (Nomaler et al., 2021). Autonomous means that demand is independent of current income, but it may still be endogenous in the model. For example, autonomous demand will depend on financial wealth of households in our conceptual model, which makes it endogenous (and also underlines the SFC nature of our model), but this type of demand does not depend on income in the current period. We will refer to demand that depends on current income (e.g., a fixed propensity to consume out of wage income that is smaller than 1) as induced demand.

The supermultiplier literature (e.g., Freitas and Serrano, 2015) proposes that investment demand is also induced demand. Here it is assumed that investment is a fixed fraction of current output (GDP). This fixed fraction itself will vary over time, where the most common modelling choice is to make it dependent on capacity utilization. We will follow this approach to the letter in our conceptual model, where we need to note that investment is a fixed fraction of gross output, not value added (this distinction does not matter in most of the supermultiplier literature, that does not consider input-output linkages). By including the investment fraction of output in the multiplier, its magnitude will increase (hence the term supermultiplier).

A large part of the supermultiplier literature focuses on the determinants of autonomous demand. Freitas and Serrano (2015) and Pariboni (2016) propose that autonomous demand is credit-financed. Other ideas are that autonomous demand is capitalists' consumption (Lavoie, 2016), exports (Nah & Lavoie, 2017), government expenditures (Allain, 2015). A common idea taken from the SFC literature (e.g., Brochier & Silva, 2019) is that autonomous demand depends on household wealth. Caminati & Sordi (2019) and Deleidi & Mazzucato (2019) have proposed government R&D as the autonomous expenditure mechanism.

An important issue is whether autonomous demand is considered as an exogenous variable, or to some extent as endogenous. With purely exogenous autonomous demand, as in Freitas and Serrano (2015), output growth will follow the growth rate of exogenous autonomous demand. Whether employment is anywhere near full employment will then purely depend on adaptation of the labour force to employment demand as it results from the exogenous growth path. This is the case in the supermultiplier model of Serrano and Freitas (2015), who consider full employment a non-issue.

### 2.4. A synthesis

If autonomous demand is endogenized, the employment rate re-enters the picture, which is a key point of our earlier contribution in Nomaler et al. (2021), and also in the conceptual model presented here. Nomaler et al. (2021) combine this with semi-endogenous R&D-based productivity growth and show that this combination leads to the emergence of a steady state growth rate that is stable under a wide range of parameter values, and which keeps the (un)employment rate fixed. In this steady state, growth is essentially supply-based R&D and productivity growth), but the demand side adjusts, and the multiplier remains the primary mechanism of economic coordination. This kind of steady state is what we will seek in the conceptual model developed here.

In this sense, our proposed conceptual model is deeply rooted in the Keynesian tradition, i.e., what Ciarli and Savona call the EKM tradition. But we also deviate from this tradition, mainly by making the steady state in which demand adapts to supply a central part of the model. This makes our model similar to those in the evolutionary (ABM) tradition, which are supply-based models that have a large role of R&D and technological progress. Dynamic input-output modelling, which is our main way of operationalizing the supermultiplier, are also an important part of our approach.

## 3. The input-output structure

The economic accounts of the model revolve around a multi-regional input-output (MRIO) table. To illustrate the equations, the accounting logic and the steady state definitions empirically, we constructed a small empirical MRIO table that consists of six countries or country blocks (EU-27, China, United Kingdom, Japan, USA and Rest-of-the-World, ROW), and 14 sectors. The sectors are listed in Table 1. This MRIO is constructed from the ExioBase database (v 3.9)[1] by aggregating the industry-by-industry version of Exiobase to the $6 \times 14$ format.

The basic structure of the MRIO table is schematically illustrated in Diagram 1. Parts of this table are colour-coded to mark their function. Labels are in light blue. With $6 \times 14 = 84$, The matrix of intermediate deliveries, which is in green, records deliveries from the row to the column, where rows and columns represent the same set of "country-sectors" (i.e., sectors within countries), of

[1] https://doi.org/10.5281/zenodo.15689391 and Stadler et al. (2025).

which there are $6 \times 14 = 84$. Thus, the matrix of intermediate deliveries is $84 \times 84$. Entries in the matrix of final demand, which is in pink, represent deliveries from the country sectors to consumers (consumption goods), to firms (gross fixed capital formation, or investment) and to government consumption. Thus, there are $6 \times 3 = 18$ columns in this part of the table, and 84 rows. Each row contains deliveries by the country-sector and hence adds up to gross output, which is a column-vector displayed in grey.

**Table 1. The 14 sectors used in the empirical model**

| | CODES | SHORT DESCRIPTION |
|---|---|---|
| **1** | A | Agriculture, forestry and fishing |
| **2** | B | Mining and quarrying |
| **3** | C | Manufacturing |
| **4** | D-E | Electricity, gas, steam; water supply, sewerage, waste management |
| **5** | F | Construction |
| **6** | G | Wholesale and retail trade; repair of motor vehicles and motorcycles |
| **7** | H & J | Transportation and storage, Information and communication |
| **8** | I | Accommodation and food service activities |
| **9** | K | Financial and insurance activities |
| **10** | L | Real estate activities |
| **11** | M-N | Professional, scientific and technical activities; administrative and support service activities |
| **12** | O-Q | Public administration, defence, education, human health and social work activities |
| **13** | R-S | Arts, entertainment, recreation; other services and service activities, etc. |
| **14** | T | Activities of households as employers; |

The yellow part of the table holds taxes minus subsidies, and is often negative, which means subsidies are larger than taxes. This contains one just row and 84 columns. We will use these numbers to calculate a fixed and possibly negative value added tax rate, which will be specified below when we present the model equations. The purple part holds value added in the 84 country-sectors (columns). This is split into two parts: wages paid to workers and the gross operating surplus (profits). Hence there are two rows in this part of the table.

A basic principle of the input-output logic is that the first 84 rows of the table (output) will add up to the same values as the sums of the first 84 columns of the table. This is represented by the fact that there is also an output row at the bottom of the table, which is colour-coded in the same way as the last column (gross output). Although there may be data in the white parts of the table, we do not need those for the exposition and the analysis of the conceptual model, hence we ignore those parts here.

Diagram 1 outlines the basis structure of the data as it comes from the MRIO (per year). In our supermultiplier model, parts of final demand will be endogenized, and hence the structure of Diagram 1 will be modified to account for the (super)multiplier. This will be explained in the next section. For this we need to make assumptions about induced consumption, and about the so-called capital flow matrix. The latter is an $84 \times 84$ matrix that works on the same principles as the matrix of intermediate deliveries. It represents deliveries of investment goods from the row sector to the column sector.

The Exiobase MRIO does not have a capital flow matrix. For the eventual model, a capital flow matrix needs to be constructed. For the conceptual model (i.e., the empirical analysis here), we

assume that all columns of the capital flow matrix for a single country have identical structure, and that this structure is equal to the structure of the GFCF column of the country in the final demand block. This is a somewhat reasonable assumption because the row-sums for a single (column) country in the capital flow matrix is equal to the GFCF vector in the final demand matrix for the country.

**Diagram 1. Schematic representation of the MRIO table**

| | 84 country-sectors | 6 countries x 3 categories (HH cons., GFCF, Gov. cons.) | Gross Output |
|---|---|---|---|
| 84 country-sectors | Intermediate deliveries | Final demand | |
| Taxes minus subsidies | | | |
| Value added (wages and gross operating surplus) | | | |
| Gross output | | | |

We constructed a time series of these MRIO tables, for the period 2007 – 2020, but we will mostly work with the 2019 table (2020 is considered to be an exceptional year because of Covid-19). The tables come in current prices, but we need versions in constant prices (i.e., real terms). For this, we use input-output tables from the Asian Development Bank (ADB)[2], which come in constant and current prices. ADB constructs the tables in constant prices by deflating each row by the same value of the deflator, hence we can re-construct the deflator and use it on our tables from Exiobase (with 2010 as the base year). This deflation procedure is identical to what we propose below in the theoretical specification of the model. In addition, we use data on employment (hours worked) from Exiobase.

## 4. The model: equations

### 4.1. Production costs, prices and the MRIO table in real terms

The model specifies relations both in nominal and real terms, where we understand "nominal" to refer to quantities or transactions in prices of the current period, and "real" to refer to quantities or transactions in the prices of a base year, which we set to 1 for all country-sectors. In the empirical illustration of the model, 2010 is the base year, i.e., all prices are 1 in 2010, and the table in nominal terms if equal to the table in real terms for that year.

All country-sectors have their own price, which they charge to all buyers, i.e., domestic and foreign buyers and buyers of all categories (intermediates and all types of final goods). From the buyer-perspective, output of a given sector can be bought from different suppliers, which we take to be represented by their location (i.e., country) in this model. Thus, in the context of the $6 \times 14$ empirical application, a buying country-sector that wants to buy manufacturing output can source this from 6 different suppliers, including one domestic supplier.

There are three types of goods that can be bought: intermediates, investment goods and final consumption goods. For all of these, we will assume that the selection of a supplier (country) is described by the well-known Armington approach (Armington, 1969; Anderson, 1979), which

[2] https://kidb.adb.org/globalization

assumes that the different suppliers (countries) of products of sector $k$ sell imperfect substitutes to the using country-sector $j$ with elasticity of substitution $1/(1-\rho_{kj})$ for intermediate goods, and with a similar elasticity to be specified below for consumption and investment goods.

We assume that production takes place with a fixed coefficients technology, which dictates that a specific amount (quantity) of intermediate inputs of each type (sector) and a fixed amount of labour must be used. We will specify the capital requirements later. For the intermediate inputs, country-sector $j$ must use an amount $b_{kj}^{sect}$ of sector $k$ output. Armington quantity shares determine the amount of these intermediates bought from each supplier $i$:

$$b_{ij} = b_{kj}^{sect} \frac{\alpha_{ij} p_i^{-\rho_{kj}}}{\sum_z \alpha_{zj} p_z^{-\rho_{kj}}}$$

where $z$ indexes the set of all country-sectors where the sector is $k$. Matrix $\mathbf{B}^{sect}$ $(n_s \times n_{cs})$ holds the elements $b_{kj}^{sect}$ and $\alpha_{ij}$ a set of positive share parameters ($\sum_z \alpha_{zj} = 1$), and $\rho_{kj}$ the Armington price elasticity of intermediate products $k$ in the country sector $j$. We assume that the value of all $\alpha$s is known at the start of the period but may vary over time due to forces that will remain outside the scope of this paper. In the empirical application, we will use time-varying values for the $\alpha$s such that the model outcome mimics the empirically observed values.

Prices result from unit variable costs (wages and intermediates) and a markup over unit costs, with $m_j$ denoting the markup in country-sector $j$. With $\mathbf{B}$ denoting the real matrix of input coefficients, i.e., element $b_{ij}$ of matrix $\mathbf{B}$ denotes the requirements of $i$-type inputs for producing 1 (real) unit of $j$-type output, the per-unit intermediate costs for 1 unit of $j$-type output are $\sum_i p_i b_{ij}$. Note that with the Armington quantity shares applied to $b_{kj}^{sect}$, $b_{ij}$ with the sector in country-sector $i$ being equal to $k$ depends on the prices of all suppliers of products $k$. For now, we will ignore this endogeneity of $b_{ij}$ and proceed as if $b_{ij}$ was known at the start of the period. We will relax this assumption when we are ready to solve for the prices $p_i$.

The labour coefficient (amount of labour needed to produce 1 unit of output) for country-sector $j$ are denoted by $g_j$, and we also assume that each country-sector has its own specific wage rate, denoted by $w_j$. This means that the labour costs for producing 1 unit of $j$-type output are $w_j g_j$, and thus total costs for producing 1 unit of $j$-type output are $\sum_i p_i b_{ij} + w_j g_j$

The government raises a tax on and possibly subsidizes production, resulting in the "taxes minus subsidies" row in the MRIO table. We assume that this is a value added tax, i.e., firms do not pay the tax on the intermediate inputs that they buy. The sum of this value added tax and subsidies is specified as a fraction of value added: $\tau_j$.[3] With this we can specify the following equation for the price in country-sector $j$:

$$p_j = (1 + m_j + \tau_j)\left(\sum_i p_i b_{ij} + w_j g_j\right)$$

Setting this in matrix notation and solving for prices, we obtain

$$\mathbf{p}^{\mathrm{T}} = \mathbf{w}^{\mathrm{T}} \widehat{\mathbf{G}} \widehat{\widehat{\mathbf{m}}} \left(\mathbf{I} - \mathbf{B} \widehat{\widehat{\mathbf{m}}}\right)^{-1}$$

[3] Alternatively, the value added tax rate can be expressed as a fraction of total production costs, as follows: $\tau_j^* = \tau_j \left(\frac{m_j(\sum_i p_i b_{ij} + w_j g_j) + w_j g_j}{\sum_i p_i b_{ij} + w_j g_j}\right)$.

where **p** is a column-vector of prices ($n_{cs} \times 1$) with $n_{cs}$ the number of country-sectors (in our empirical application: 84), the superscript $^{T}$ denotes transposition, $\hat{\hat{\mathbf{m}}}$ is a matrix with $1 + m_j(1 + \tau_j)$ on the main diagonal and zeros elsewhere, $\hat{\hat{\hat{\mathbf{m}}}}$ is a matrix with $(1 + \tau_j)(1 + m_j)$ on the main diagonal and zeros elsewhere, **I** is the identity matrix, **w** is the column vector of wage rates ($n_{cs} \times 1$), and $\hat{\mathbf{G}}$ is a matrix with the labour coefficient of the country-sectors on the main diagonal and zeros elsewhere. All matrices in this equation are $n_{cs} \times n_{cs}$.

We assume that at the beginning of every period, wage rates as in **w**, labour coefficients as in $\hat{\mathbf{G}}$, the markups $m_j$, tax rates $\tau_j$, and hence the matrices $\hat{\hat{\mathbf{m}}}$ and $\hat{\hat{\hat{\mathbf{m}}}}$ are known and fixed (they result from the previous period). However, the elements of matrix **B** depend on the price vector **p**, as explained above. Thus, **p** appears on both sides of the last equation: we must now relax the assumption of an exogenous matrix **B**.

We choose to solve the above equation numerically to determine prices. We supply a Python script that implements this numerical procedure. This is explained in the Python Box below, which uses the price algorithm in a calculation of the short-run solution for 2019.

Using these numerically solved prices **p**, the MRIO table can be deflated in the usual way, i.e., the first $n_{cs}$ rows of the table are divided by the corresponding price level. The column-total row for taxes minus subsidies and the two value added rows is derived as a residual (between deflated gross output in the column and the sum of deflated intermediate inputs). The relative shares of taxes minus subsidies, total wages and the operating surplus remains identical to the shares in the nominal table.

### 4.2. Consumption

The core of the (short-run) determination of output is a closed input-output system, in which both consumption and investment are part of a multiplier system. We start by specifying the consumption process. Consumption has both an induced part and an autonomous part. The induced part depends on current period income, in particular labour income. We chose to ignore any dependence of induced consumption on profit income (i.e., the gross operating surplus), but this could be incorporated into the model without changing the basic idea.

With employment and labour income denoted by the $n_{cs} \times 1$ column vectors **L** and **y**, respectively, we have

$$\mathbf{y} = \hat{\mathbf{w}}\mathbf{L}$$

where $\hat{\mathbf{w}}$ is a matrix with the wage rates on the main diagonal and zeros otherwise. Note that vector **y** is observed in the nominal MRIO table as part of value added.

Workers in country-sector $i$ have marginal propensity to consume out of wage income equal to $c_i < 1$. These are collected in the diagonal $n_{cs} \times n_{cs}$ matrix $\hat{\mathbf{c}}$. With the amount of (wage) income of workers in country-sector $j$ determined, this is distributed over the suppliers of consumption goods in a way that is similar to the way the matrix of input coefficients **B** is determined. This means that for every $ that workers of country-sector $j$ spend on consumption, they will spend

$$r_{ij} = r_{kj}^{sect} \frac{\beta_{ij} p_i^{1-\varrho_{kj}}}{\sum_z \beta_{zj} p_z^{1-\varrho_{kj}}}$$

on products supplied by country-sector $i$, where $r_{kj}^{sect}$ is the share spent on products $k$ offered by all suppliers (countries) of $k$, and $k$ is the sector of the country-sector $i$, $\alpha_{ij}$ a set of positive share parameters ($\sum_z \alpha_{zj} = 1$), and $\varrho_{kj}$ the Armington price elasticity of consumption goods $k$ in the country sector $j$. Note that because consumers spend a fraction of their nominal income rather than buying a fixed (real) quantity, the above equation specifies expenditure shares (rather than quantity shares in the case of intermediate products), as indicated by the exponent $1 - \varrho_{kj}$ instead of $-\varrho_{kj}$ (the latter applies in the case of quantity shares).

We assume that the values $r_{kj}^{sect}$, which make up matrix $\mathbf{R}^{sect}$, are given at the beginning of each period, although they may change over time due to factors that will remain outside the scope of this paper (like $\mathbf{B}^{sect}$). Then with prices $\mathbf{p}$ already known, all values $r_{ij}$ are also known. These values $r_{ij}$ are the elements of the $n_{cs} \times n_{cs}$ matrix $\mathbf{R}$ that distributes consumption spending over the supplying country-sectors. Columns in matrix $\mathbf{R}$ sum to 1. Then define the new matrices

$$\mathbf{C} \equiv \mathbf{R}\hat{\mathbf{c}} \text{ and } \mathbf{K} = \hat{\mathbf{p}}^{-1}\mathbf{C}\hat{\mathbf{w}}$$

In matrix $\mathbf{C}$, column $j$ adds up to $c_j$, and the element $c_{ij}$ is equal to the amount of spending per \$ income by workers of $j$ on products produced in $i$.[4] Every unit of labour that is employed earns the going wage rate, which implies that the $n_{cs} \times n_{cs}$ matrix $\mathbf{C}\hat{\mathbf{w}}$ specifies the distribution of induced consumption spending per unit of labour (employed in the column country-sectors). The matrix $\mathbf{K}$ converts this to real terms.

Apart from this, there is an autonomous part of consumption, which at this stage we will consider as exogenous, but for which we will propose an endogenization below. Autonomous consumption by households is denoted by the matrix $\mathbf{\Phi}$, which is $n_{cs} \times n_c$, i.e., autonomous consumption spending is aggregated over all households (instead of being split by the sector of employment of the households). In this matrix the element $\phi_{ij}$ denotes nominal autonomous demand supplied by country-sector $i$ and consumed in country $j$. In addition, we allow for nominal autonomous government demand, denoted by the matrix $\mathbf{\Phi}^{\boldsymbol{g}}$. Then we construct the $n_{cs} \times 1$ vector $\mathbf{f} = (\mathbf{\Phi} + \mathbf{\Phi}^{\boldsymbol{g}})\mathbf{1}$, where $\mathbf{1}$ is an $n_{cs} \times 1$ vector of 1s. Total autonomous demand by supplying sector is then equal to $\hat{\mathbf{p}}^{-1}\mathbf{f}$.

### 4.3. Investment

To start the description of the productive structure of the economy, we introduce the variable $\kappa^{pc}$, which is the capital stock of a country-sector, measured in productive capacity ($pc$) units. For measurement in productive capacity units, we introduce the "normal" capital-to-output ration in a country-sector $j$, denoted by $\nu_j$. Thus, every unit of capital is expected to produce $1/\nu_j$ units of output, and if firms in the country-sector $j$ wish to invest an amount equivalent to 1 unit of productive capacity, they should buy (invest in) $\nu_j$ units of capital (disregarding depreciation of existing capital).

The $n_{cs} \times n_{cs}$ matrix $\mathbf{N}^{base}$ with elements $n_{ij}^{base}$ specifies the composition of a unit of productive capacity in the country-sector of the column ($j$). The sum of the column is equal to $\nu_j$ and the

[4] Note that we differentiate consumption preferences in $\mathbf{R}$ only between the countries (as opposed to the country-sectors). This means that the columns of matrix $\mathbf{R}$ that corresponds to the $n_s$ sectors of a given country are going to be identical. In the empirical application, we will use the household final demand columns to specify $\mathbf{R}$.

elements of the column represent the contributions of all country-sectors together to the capital-output ratio. Thus, if country-sector $j$ wants to expand productive capacity by one unit, it buys $n_{ij}^{base}$ quantity units from every country-sector $i$.

As in the case of the matrices **B** and **R**, matrix $\mathbf{N}^{base}$ results from a sectoral component (represented by $\mathbf{B}^{sect}$ and $\mathbf{R}^{sect}$ for **B** and **R**, respectively) and the application of Armington shares to allocate the demand for (investment) goods produced in sector $k$ to suppliers of products $k$. In this case, like in the case of **B**, we use Armington quantity shares, because investing firms aim to buy a fixed quantity of productive capacity. Thus, we have an $n_s \times n_{cs}$ matrix $\mathbf{N}^{baseSect}$ with elements $n_{kj}^{baseSect}$ that specify the sectoral composition of a unit of productive capacity in country-sector $j$. In this matrix, the column $j$ sums to $v_j$, and if we divide all elements in the column by $v_j$, the column sum will be 1. As in matrix $\mathbf{N}^{base}$, the columns of matrix $\mathbf{N}^{baseSect}$ sum to $\eta_j$. With the Armington quantity shares, the elements of matrix $\mathbf{N}^{base}$ are equal to

$$n_{ij}^{base} = n_{kj}^{baseSect} \frac{\gamma_{ij} p_i^{-\theta_{kj}}}{\sum_z \gamma_{zj} p_z^{-\theta_{kj}}}$$

where $k$ is the sector of the country-sector $i$, $\gamma_{ij}$ a set of positive share parameters ($\sum_z \gamma_{zj} = 1$), and $\theta_{kj}$ the Armington price elasticity of investment goods $k$ in the country sector $j$.

Following the Sraffian supermultiplier logic, country-sectors invest a fraction $h$ of their gross output (with both investment and output in real terms). Thus, with $\hat{\mathbf{h}}$ the matrix with the investment propensities $h$ on the main diagonal and zeros otherwise, $\mathbf{N} = \mathbf{N}^{base}\hat{\mathbf{h}}$ is the $n_{cs} \times n_{cs}$ investment flow matrix that works in a way similar to the matrix of input coefficients **B**. Whereas **B** specifies intermediate inputs needed per unit of output of the column, **N** gives investment needed per unit of output of the column (both in real terms). Matrix $\mathbf{N}\hat{\mathbf{Q}}$ (with $\hat{\mathbf{Q}}$ the matrix with gross output on the main diagonal and zeros otherwise) is the $n_{cs} \times n_{cs}$ investment matrix, with rows as suppliers of investment goods, and columns as users. In the traditional ("open") input-output table, this matrix belongs in the final demand block, where it is usually found in $n_{cs} \times 1$ format (i.e., all columns aggregated into one).

### 4.4. Short-run closure of the model

To close the model, we stack the $n_{cs} \times 1$ vectors of gross output **Q** and employment **L**, and using the relationships and definitions specified so far, the following equation sets up the basic calculations leading to output and employment:

$$\begin{bmatrix}\mathbf{Q}\\ \mathbf{L}\end{bmatrix} = \begin{bmatrix}\mathbf{B}+\mathbf{N} & \mathbf{K}\\ \hat{\mathbf{G}} & \mathbf{0}\end{bmatrix}\begin{bmatrix}\mathbf{Q}\\ \mathbf{L}\end{bmatrix} + \begin{bmatrix}\mathbf{f}\\ \mathbf{v_0}\end{bmatrix}$$

where $\mathbf{0}$ is a $n_{cs} \times n_{cs}$ matrix with only 0s, and $\mathbf{v_0}$ is a $n_{cs} \times 1$ vector of 0s.[5] Note that $\begin{bmatrix}\mathbf{B}+\mathbf{N} & \mathbf{K}\\ \hat{\mathbf{G}} & \mathbf{0}\end{bmatrix}$ is a compartmentalized matrix of four different $n_{cs} \times n_{cs}$ matrices. This can be solved in the usual way, leading to

$$\begin{bmatrix}\mathbf{Q}\\ \mathbf{L}\end{bmatrix} = \left[\begin{bmatrix}\mathbf{I}-\mathbf{B}-\mathbf{N} & -\mathbf{K}\\ -\hat{\mathbf{G}} & \mathbf{I}\end{bmatrix}\right]^{-1}\begin{bmatrix}\mathbf{f}\\ \mathbf{v_0}\end{bmatrix}$$

[5] Instead of **0**, a vector of exogenous (and autonomous) income, such as unemployment benefits, could be inserted here. This will be briefly discussed below.

The process in which this solution is calculated is illustrated graphically in Diagram 2. The solution in terms of the above equation is very similar to the solution obtained for the open input-output model. For that model, we would have the inverse Leontief matrix $(\mathbf{I}-\mathbf{B})^{-1}$, which acts as a multiplier matrix. In the open model, output results from multiplying the inverse Leontief multiplier matrix with exogenous final demand. In our open model, the multiplier is both larger and slightly more complicated. In addition to multiplier effects due to intermediate deliveries, $(\mathbf{I}-\mathbf{B})^{-1}$ in the closed model), our open model has multiplier effects due to investment demand (the Sraffian supermultiplier logic leading to inclusion of $\mathbf{N}$ in the multiplier matrix) and a Keynesian income multiplier (the inclusion of $\mathbf{K}$ and $\widehat{\mathbf{G}}$ in the multiplier matrix).

The multiplier matrix has a clear economic interpretation. In the open model case, an element $l_{ij}$ in the inverse Leontief matrix $(\mathbf{I}-\mathbf{B})^{-1}$ corresponds to the increase of gross output in $i$ due to an increase of final demand by 1 unit in $j$. The sum $\sum_j l_{ij}$ is the (backward) multiplier of column $j$: the total (global) increase of gross output due to an increase of final demand by 1 unit in $j$.

In our closed model, the upper-left $n_{cs} \times n_{cs}$ block in the multiplier matrix $\begin{bmatrix}\mathbf{I}-\mathbf{B}-\mathbf{N} & -\mathbf{K}\\ -\widehat{\mathbf{G}} & \mathbf{I}\end{bmatrix}^{-1}$ has a similar interpretation. Each element of this block corresponds to the increase of gross output in $i$ due to an increase of autonomous demand by 1 unit in $j$. But these multipliers are larger than in the inverse Leontief matrix $(\mathbf{I}-\mathbf{B})^{-1}$ of the open model because of the Sraffian supermultiplier (the inclusion of $\mathbf{N}$).[6] In the bottom-left $n_{cs} \times n_{cs}$ block of the multiplier matrix, we find what we call employment multipliers: an element $x_{ij}$ in this part of the multiplier matrix corresponds to the increase of employment in $i$ due to an increase of final demand by 1 unit in $j$. In the righthand side half of the multiplier matrix, similar interpretations exist, but here we do not evaluate changes due to an increase of autonomous demand but rather changes due to an increase in exogenous and autonomous incomes that could be inserted into $\begin{bmatrix}\mathbf{f}\\ \mathbf{v_0}\end{bmatrix}$ in lieu of $\mathbf{v_0}$.

[6] It should be kept in mind that by inverting using Schur complements, in the inverse matrix $\begin{bmatrix}\mathbf{I}-\mathbf{B}-\mathbf{N} & -\mathbf{K}\\ -\widehat{\mathbf{G}} & \mathbf{I}\end{bmatrix}^{-1}$ each of the four blocks depend on all constituent matrices in $\begin{bmatrix}\mathbf{I}-\mathbf{B}-\mathbf{N} & -\mathbf{K}\\ -\widehat{\mathbf{G}} & \mathbf{I}\end{bmatrix}$.

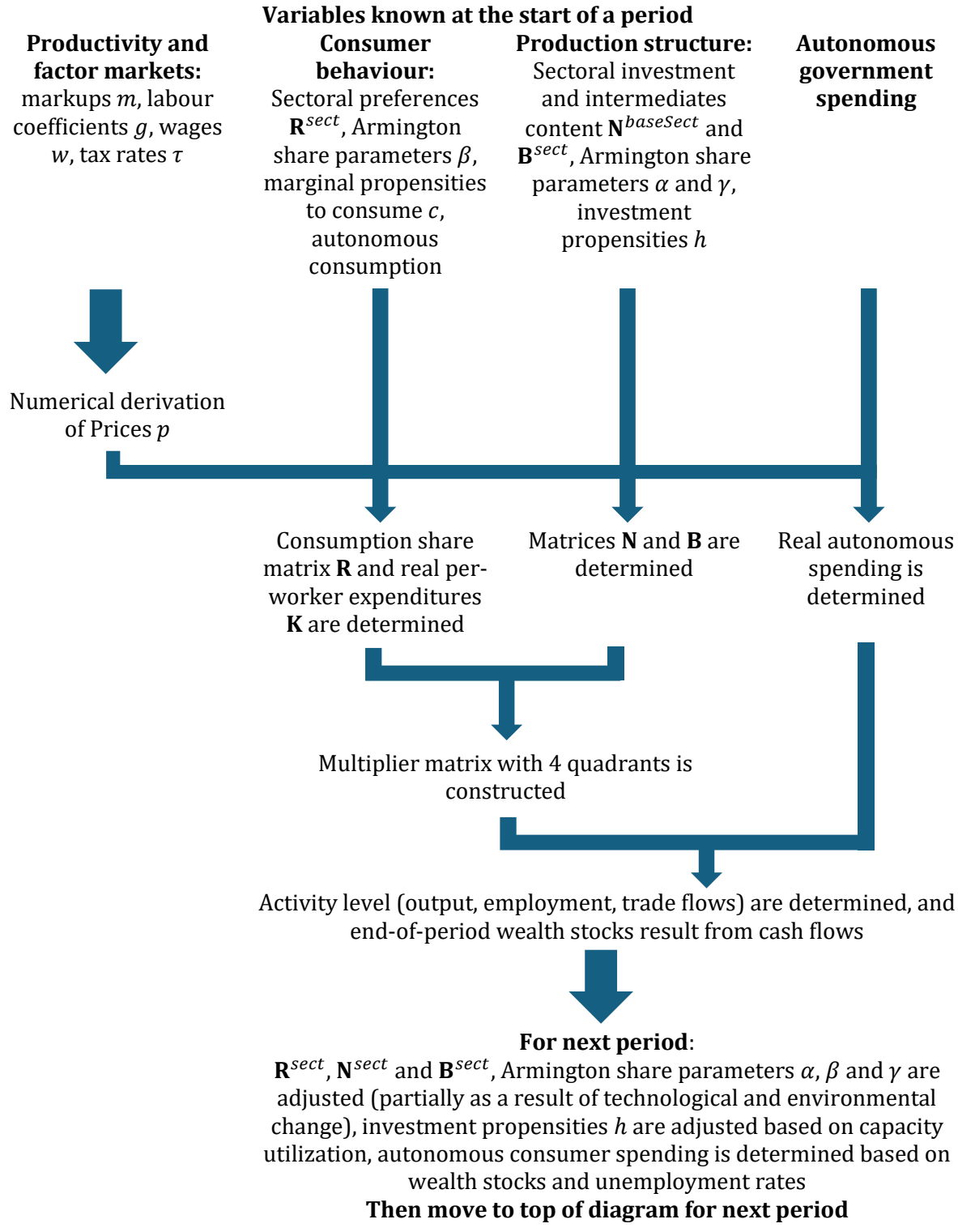



**Diagram 2. Solution flow for short-run model**

**Python Box – The multiplier matrix and the short-run solution of the model**

We provide and implement a short Python script (in the form of a Jupyter notebook called *CalculateModel2019.ipynb*) that calculates the short-run solution of the model for our 2019 $6 \times 14$ empirical setting. The script calculates output and employment levels using the short-run model's equations. As a default, it produces actual output and employment levels in 2019. But we also run a little experiment in which we raise 2019 productivity in EU27 sectors by 10% and compare the result to the actual real-world outcome.

In order to implement these calculations, we need a few additional parameters and model changes. First of all, we need marginal propensities to consume out of wages, for each of the six countries. We derive those values from Khan et al. (2015), Table 5 (first column, LnGDP). The values we used are: 0.557 for EU27 (average of France, Germany and Italy), 0.62 for China, 0.87 for UK, 0.42 for Japan, 0.51 for USA, and 0.595 (average of the 5 other values) for ROW.

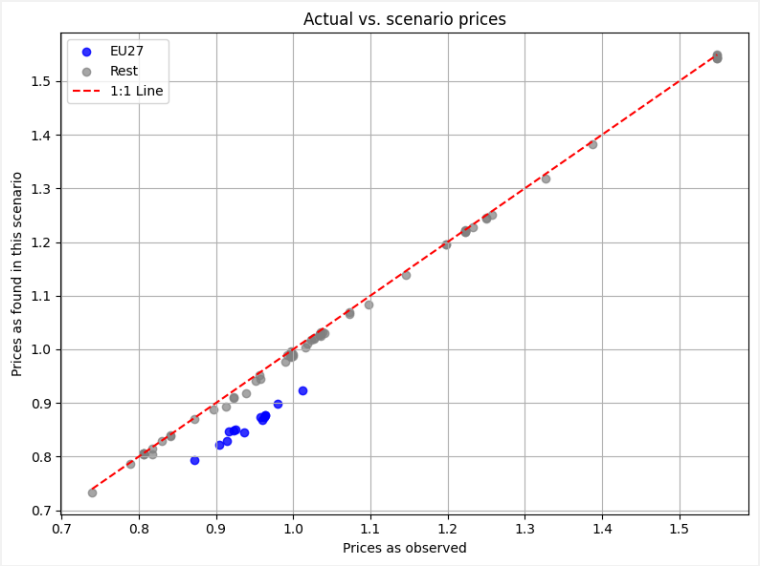
We need some model changes and parameters related to the role of the government in the real-world data. There is sizable government final demand, which we treat as exogenous and autonomous, and add to real valued autonomous demand. There are also income taxes (used to cover government expenditures) that are different from the value-added tax rate that we use in the theoretical model. A new variable $inctax_i$ is introduced to represent this, where $i$ is a country. The only role of this new variable is that we multiply the propensities to consume out of wages, $c_i$ by $(1 - inctax_i)$. We calibrate the values of $inctax_i$ to 0.0703 for EU27, 0.0307 for China, 0.117 for UK, 0.0935 for Japan, 0.0931 for USA and 0.061 for ROW. These values are derived from the World Development Indicators database, by dividing "Taxes on income, profits and capital gains" by GDP.

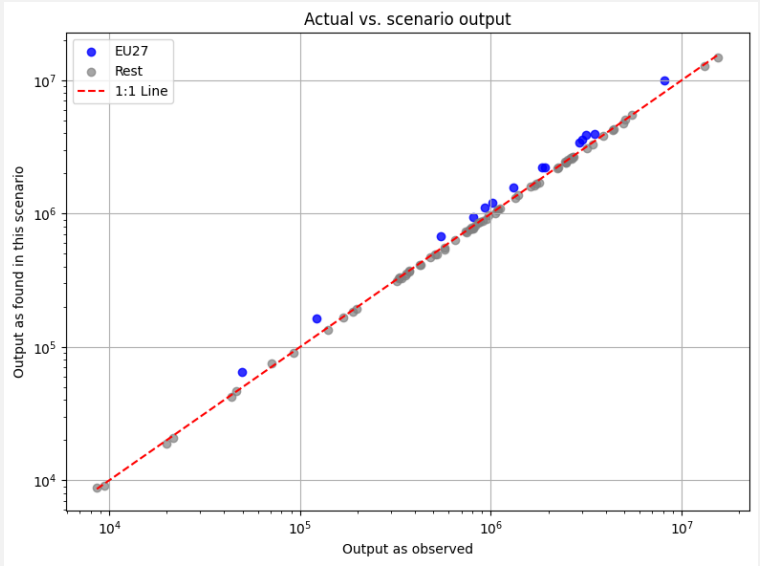
We set $\rho = \varrho = \theta = 3.8$, which is the median value reported in Bajzik et al. (2020). We calculate all $\alpha$, $\beta$ and $\gamma$ such that in combination with actual prices they generate bilateral trade flows (of all kinds of goods) equal to those in the MRIO. We also $\mathbf{N}^{baseSect}$, $\mathbf{R}^{sect}$ and $\mathbf{B}^{sect}$ in accordance with the real-world data. All of this ensures that the outcome of the short-run calculations is equal to the values found in the MRIO table, including employment.

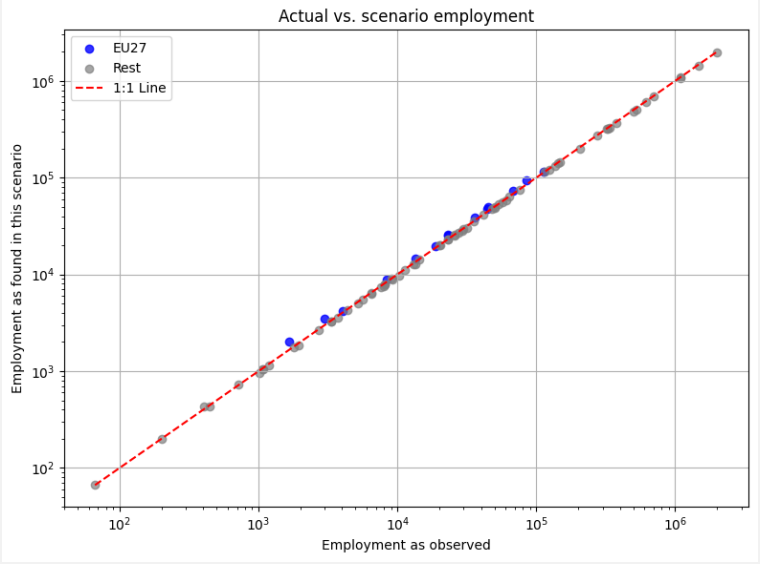
The Python script calculates the price level by numerical optimization and then applies the model's equations to derive output and employment. If the productivity shock is implemented, labour coefficients for the EU27 in matrix $\widehat{\mathbf{G}}$ fall.

In order to visualize the differences between the real-world outcome (no productivity shock) and the outcome with the productivity shock, we plotted prices, (real) output and employment as found in the experiment against their values in the calculations without the shock. These plots are provided below. Note that if no productivity shock would be implemented, these plots would show all data on the straight 45 degrees lines. It can be seen that the productivity shock productivity shock in EU27 makes prices fall, especially in EU27, and output and employment in EU27 increases.

## Python Box (continued) – Plotting results



### 4.5. A closer look at the multiplier matrix

We illustrate the interpretation of the multiplier matrix by taking a closer look at the empirical case for the short-run solution for 2019 that was derived in the Python Box. However, before we look at the empirical values, we provide an intuitive interpretation of the conceptual form of the multiplier matrix:

$$\begin{bmatrix} \mathbf{I}-\mathbf{B}-\mathbf{N} & -\mathbf{K} \\ -\widehat{\mathbf{G}} & \mathbf{I} \end{bmatrix}^{-1}$$

As we have already seen, this clearly shows that the multiplier matrix has four quadrants: one associated to each of the parts $\mathbf{I}-\mathbf{B}-\mathbf{N}$, $-\mathbf{K}$, $-\widehat{\mathbf{G}}$, and $\mathbf{I}$.[7] The part associated to $\mathbf{I}-\mathbf{B}-\mathbf{N}$ is somewhat similar to the inverse Leontief matrix that is familiar from the analysis of the open input-output model. In the inverse Leontief matrix, an element $(i, j)$ represents the impact of an increase of final demand for sector $j$ by \$1 on gross output in sector $i$. If we sum over a column, we obtain the backward multiplier, which is the total (global) increase of gross output as a result of a \$1 increase of final demand in the column. These backward multipliers are an important indicator of the degree to which the (column) sector is linked to the global economy.

In our case, the interpretation of this part of the multiplier matrix is similar as the idea of backward multipliers. However, in addition to the effect of intermediate linkages (which generally make backward multipliers >1 in the case of the inverse Leontief matrix), we have two effects that contribute to the multiplier. One is the effect of making the input-output model closed, or, in other words, of introducing feedback from production to labour income and from labour income to consumption. This works through $\widehat{\mathbf{G}}$ and $\mathbf{K}$ and generally increases the size of the multiplier. The other additional effect is from investment flows, which turns the multiplier into a supermultiplier: not only does additional autonomous demand induce more demand for intermediates and consumption goods, but also the demand for investment goods goes up.

The part of the multiplier matrix that is associated to $-\widehat{\mathbf{G}}$ has similar interpretation, but now the outcome variable is employment. The element $(i, j)$ in this part of the multiplier matrix represents the effect on an increase of autonomous demand by \$1 in $j$ on employment in $i$. We can call the column sums in this submatrix employment multipliers.

The parts associated to $-\mathbf{K}$ and $\mathbf{I}$ again have similar interpretations. Here we see the impact of autonomous income, such as unemployment benefits or other social security, in the column on output ($-\mathbf{K}$) or employment ($\mathbf{I}$). We will not dwell on this here, but this may be an interesting option to explore further in a more final version of the model.

There is one final characteristic that we must explain about the multiplier matrix. This is related to the analysis of global value chains (GVCs). In input-output related GVC analysis, we generally use an open but multi-regional input output (MRIO) table, and we start by calculating the inverse Leontief matrix of the system. Then we calculate the value added shares of gross output in each column of the MRIO and represent them in a diagonal matrix (value added shares on the main diagonal, zeros elsewhere). Then we pre-multiply the inverse Leontief matrix with this diagonal value added share matrix.

In the resulting matrix, columns will add up to 1 and the entire column provides a backward decomposition of value added delivered by the column sector. For example, if we look at the column for transport equipment in Germany, then the column entries show the relative contribution of all sectors worldwide to the final demand that is delivered by this country-sector.

[7] Once again, it must be noted that these parts are not equal to those expressions, because the multiplier matrix is the inverse of the entire matrix in square brackets.

For example, we may find that 25% of the value of final demand delivered is provided by the Japanese steel industry, 35% by the Dutch electronics sector, 20% by the Chinese rubber industry and 20% by the German transport equipment sector itself.

In such an open input-output system, the columns in this "GVC matrix" must necessarily add up to 1, because in the MRIO table itself, the total value of final demand is equal to the total value of value added. However, in the multiplier matrix for our closed MRIO system, this restriction does not hold. We can pre-multiply the $\mathbf{I} - \mathbf{B} - \mathbf{N}$ associated part of the multiplier matrix with the diagonal value added share matrix and obtain the "closed" version of the GVC matrix. For the same reasons why the backward multipliers in our system are larger than the ones in the inverse Leontief matrix of the open system, the column sums of this "closed" GVC matrix will generally be >1. This means that after the pre-multiplication by the diagonal value added share matrix, we can calculate the backward value added (or GDP) multipliers that provide insights into the working of GVCs.

This is illustrated in Figure 1, which documents the backward value added multipliers for all country-sectors in our empirical illustration (2019). We split the total value added multipliers into a domestic part (capturing all rows that refer to the same country as the column) and a foreign part (all other rows). The average value is 2.195, clearly above 1. As it turns out the value of the Chinese value added multipliers are high as compared to the other countries, especially in real estate (L). This is a tendency that is already present in the open MRIO backward multipliers.

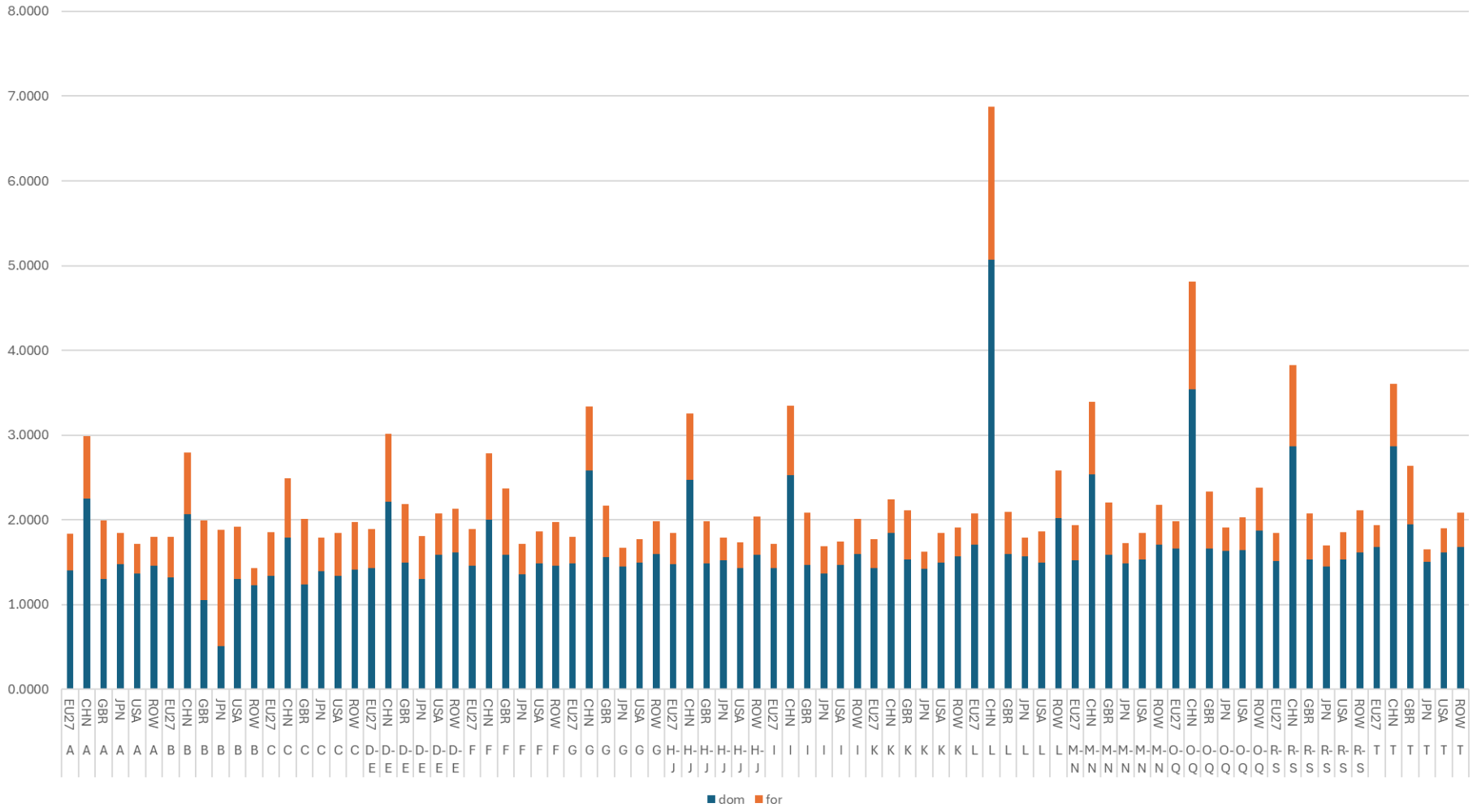



**Figure 1. Value added multipliers for the 2019 empirical implementation of the model**

### 4.6. Intertemporal dynamics

A dynamic version of the model is created when some of the variables that are given in the short-run solution of the model are changed between periods. In the next section, we will illustrate a preliminary version of such a dynamic model by using the dynamic relationships that we will outline now. These propose dynamic equations bring out the full extent of the supermultiplier logic and the stock-flow consistent nature of the model.

*The investment accelerator*

We start with the supermultiplier logic, which dictates that the matrix $\hat{\mathbf{h}}$ responds to capacity utilization. We specify a perpetual inventory process for the capital stock $\kappa^{pc}$, with $\delta$ as the depreciation rate:

$$\kappa_{t,i}^{pc} = (1 - \delta_i)\kappa_{t-1,i}^{pc} + h_{i,t}Q_{i,t}$$

Note that by our adoption of the supermultiplier principle, a fraction $h$ of gross output is invested: $h_i Q_i = I_i^{pc}$.

The $h_{i,t}$ values are updated at the beginning of each period $t$, following the usual SSM accelerator dynamics as

$$h_{i,t} = h_{i,t-1}\left[1 + \xi\left(\frac{Q_{t-1,i}}{\kappa_{t-1,i}^{pc}} - \bar{u}_i\right)\right]$$

where the adjustment rate $\xi$ and the target capacity utilization rate $\bar{u}_i$ are parameters.

*Stocks and flows of debt and assets*

Transactions between firms and households do not generally balance out within a single period, which means that households as well as firms in the country-sectors will generally have financial claims on each other. In line with the stock flow consistent modelling literature (e.g., Godley and Lavoie, 2007), we summarize the way in which these claims emerge in a transaction table (Table 2) and balance sheets (Table 3) for a two-country global economy. Note that for simplicity this excludes the government sector, and that we document households and firms in a country as an aggregate (i.e., we sum over other country-sectors in a country).

These tables adopt some slightly new notation. We use $C_k^i$, $U_k^i$ and $I_k^i$ for, respectively, the value of consumption, intermediates and investment goods produced in country $i$ and sold to / used in country $k$ (these are aggregates over country-sectors and households within a country). The absence of a subscript in these variables implies an aggregate, e.g., $C^i$ denotes the value of consumption goods produced in country $i$ irrespective of the country of use. Similarly, the absence of a superscript denotes aggregation over producing countries, e.g., $U_k$ denotes the value of total use of intermediates in country $k$, irrespective of where they are produced.

The first four rows of the transactions table (Table 2) represent international trade. Households spend (hence the negative sign in this column) money on consumption, which they buy from producers in both countries. Firms sell (hence the positive sign) consumption goods to consumers in both countries. The sum of these entries for households and firms in the same country represents the contribution of consumption to the trade balance of the country, and the sum of these entries in the global economy is zero.

**Table 2. Transactions matrix for a two-country global economy**

| | **Households in $c_1$** | **Firms in 1** | **Sum 1** | **Households in 2** | **Firms in 2** | **Sum 2** | **Sum global** |
|---|---|---|---|---|---|---|---|
| Consumption | $-C_1^1 - C_1^2$ | $+C_1^1 + C_2^1$ | $-C_1^2 + C_2^1$ | $-C_2^1 - C_2^2$ | $+C_1^2 + C_2^2$ | $-C_2^1 + C_1^2$ | 0 |
| Intermediate use | 0 | $-U_1^1 - U_1^2 + U^1$ | $-U_1^2 + U_2^1$ | 0 | $-U_2^1 - U_2^2 + U^2$ | $-U_1^2 + U_2^1$ | 0 |
| Investment (gross) | 0 | $-I_1^1 - I_1^2 + I^1$ | $-I_1^2 + I_2^1$ | 0 | $-I_2^1 - I_2^2 + I^2$ | $-I_2^1 + I_1^2$ | 0 |
| Trade balance (Sum of the above) | | | $TB_1$ | | | $TB_2$ | 0 |
| Wages | $W_1$ | $-W_1$ | 0 | $W_2$ | $-W_2$ | 0 | 0 |
| Interest | $O_1^1 + O_1^2$ | $-O_1$ | $-O_2^1 + O_1^2$ | $O_2^1 + O_2^2$ | $-O_2$ | $-O_1^2 + O_2^1$ | 0 |
| Subtotal: Cash flow | $W_1 + D_1 - C_1$ | $C^1 + U^1 + I^1 - U_1 - I_1 - W_1 - D_1$ | | $W_2 + D_2 - C_2$ | $C^2 + U^2 + I^2 - U_2 - I_2 - W_2 - D_2$ | | 0 |
| Subtotal: Current account for countries | | | $TB_1 - O_2^1 + O_1^2$ | | | $TB_2 - O_1^2 + O_2^1$ | 0 |
| Issuance of (private) bonds, Capital account for country sums | $-\Delta D_1^1 - \Delta D_1^2$ | $+\Delta D^1$ | $-\Delta D_2^1 + \Delta D_1^2$ | $-\Delta D_2^1 - \Delta D_2^2$ | $+\Delta D^2$ | $-\Delta D_1^2 + \Delta D_2^1$ | 0 |
| Total | 0 | 0 | 0 | 0 | 0 | 0 | 0 |

Notes: we do not distinguish between the current and capital accounts of firms, which implies that we do not have to include depreciation of capital in the transactions table; Empty (as opposed to 0) cells are not applicable and do not contribute to row – or column totals.

**Table 3. Balance sheets in the two-country global economy**

| | **Households in $c_1$** | **Firms in $c_1$** | **Sum $c_1$** | **Households in $c_2$** | **Firms in $c_2$** | **Sum $c_2$** | **Sum global** |
|---|---|---|---|---|---|---|---|
| Capital stock | 0 | $+\kappa_{c_1}^{val}$ | $+\kappa_{c_1}^{val}$ | 0 | $+\kappa_{c_2}^{val}$ | $+\kappa_{c_2}^{val}$ | $+\kappa_{c_1}^{val} + \kappa_{c_2}^{val}$ |
| | | | | | | | |
| Private bonds | $+D_1^1 + D_1^2$ | $-D^1$ | $-D_2^1 + D_1^2$ | $+D_2^1 + D_2^2$ | $-D$ | $-D_1^2 + D_2^1$ | 0 |
| Net Worth | $-NWH_1$ | $-NWF_1$ | $-NWH_1 - NWF_1$ | $-NWH_2$ | $-NWF_2$ | $-NWH_2 - NWF_2$ | $-\kappa_{c_1}^{val} - \kappa_{c_2}^{val}$ |
| Sum | 0 | 0 | 0 | 0 | 0 | 0 | 0 |

A similar logic holds for intermediate good and investment good, but in this case, firms are users as well as producers, hence in these cases, both positive (production) and negative (using) entries within a country occur in the column for firms. Note $U^1 = U_1^1 + U_2^1 \rightarrow -U_1^1 - U_1^2 + U^1 = -U_1^2 + U_2^1$ and similarly for the other intermediate and investment entries for firms. The trade balance, denoted by $TB$, is the sum of the first three rows, e.g., for country 1 we have $TB_1 = -C_1^2 + C_2^1 - U_1^2 + U_2^1 - I_1^2 + I_2^1$. Elaborating the expression for $TB_2$ and adding up both shows that $TB_1 + TB_2 = 0$.

A first component of households' income are wages, and because we assume that workers live in the same country as they work, wages remain within country borders. Hence the sum of wages received ($+W$) by households and wages paid ($-W$) by firms is zero for each of the two countries. Note that we adopt the notation $W_i$ for wages paid/received within country $i$.

We assume that no equity is issued by firms, and therefore all debt owed by firms takes the form of bonds. We assume that households hold these bonds and that firms owe interest on them.[8] Interest payments to households can cross borders, because households can hold bonds issued by foreign firms. We denote these payments by $O_k^i$ for interest paid by firms in country $i$ and received by households in country $k$. Because of the cross-border payments of $O$, payments and receipts of $O$ do not necessarily balance within a country, i.e., they add to the current account of the balance of payments between the countries. This is reflected in the subtotal – current account row of Table 2.

Adding up income and expenditures yields the cash flow – subtotal row for households and firms, or the current account row, for countries. The sum of cash flows for households and firms in the two countries will add up to zero, but that none of these four entries individually will be zero. In other words, we expect that household and firms in the two countries generally do not have a zero cash flow. In the stock-flow logic (Godley and Lavoie, 2007), these non-zero cash flows accumulate into financial stocks that are represented by bond holdings $D$, where $D_k^i$ denotes bonds issued in country $i$ and held in country $k$.

In the transaction table, changes in the bond holding are found in the penultimate row (above the totals). A positive cash flow for households means that they accumulate financial wealth, and in order to make the column sum to zero, this is represented by a negative sign of $\Delta D$ in the household columns. On the other hand, a negative cash flow by firms (e.g., when expenditures for investment exceeds the gross operating surplus) leads to an increase of debt, which is represented by a positive sign on $\Delta D$ in the columns for firms. Aggregated within countries, these changes in bond holdings represent the capital account of the balance of payments. The model is summarized in graphical form in Diagram 3.[9]

As in Nomaler, Spinola and Verspagen (2021; 2025) we expect that except in exceptional circumstances, households will hold a positive amount of financial wealth, which implies that firms (as an aggregate) are indebted. This is reflected in the balance sheet for the two-country global economy in Table 3. This table introduces a new variety of the capital stock, denoted by $\kappa^{val}$. This is the capital stock in historical prices, which is different from the capital stock in real productive capacity units, $\kappa^{pc}$ (introduced above).

The equation for the evolution of $\kappa^{val}$ is as follows:

[8] The use of the terminology "bonds" and "interest" is notional, we may think of $D$ also as equity, and payments $O$ as dividends, or as a respective mix of the two.
[9] The diagram has been kindly prepared by Raghav Chakravarthy.

$$\kappa_{t,i}^{val} = (1 - \delta_i)\kappa_{t-1,i}^{val} + \left(\mathbf{p}_t^T \mathbf{N}_t \widehat{\mathbf{Q}}_t\right)_i$$

Note that the term $\mathbf{p}_t^T \mathbf{N}_t \widehat{\mathbf{Q}}_t$ is a $1 \times n_{cs}$ vector of which the $i$th element represents the expenditures for investment of country-sector $i$ in period $t$. Thus, this equation adds current investment expenditures (in nominal terms) to the capital stock in historical prices and subtracts a fixed share as depreciation.

We use $\kappa^{val}$ as opposed to $\kappa^{pc}$ in the balance sheets of the firms because it is more meaningful to compare the debt $B$ of firms, which is denoted in nominal terms, to $\kappa^{val}$ than to $\kappa^{pc}$. This will become clear in the next step, which is to look at the markups that firms charge over production costs.

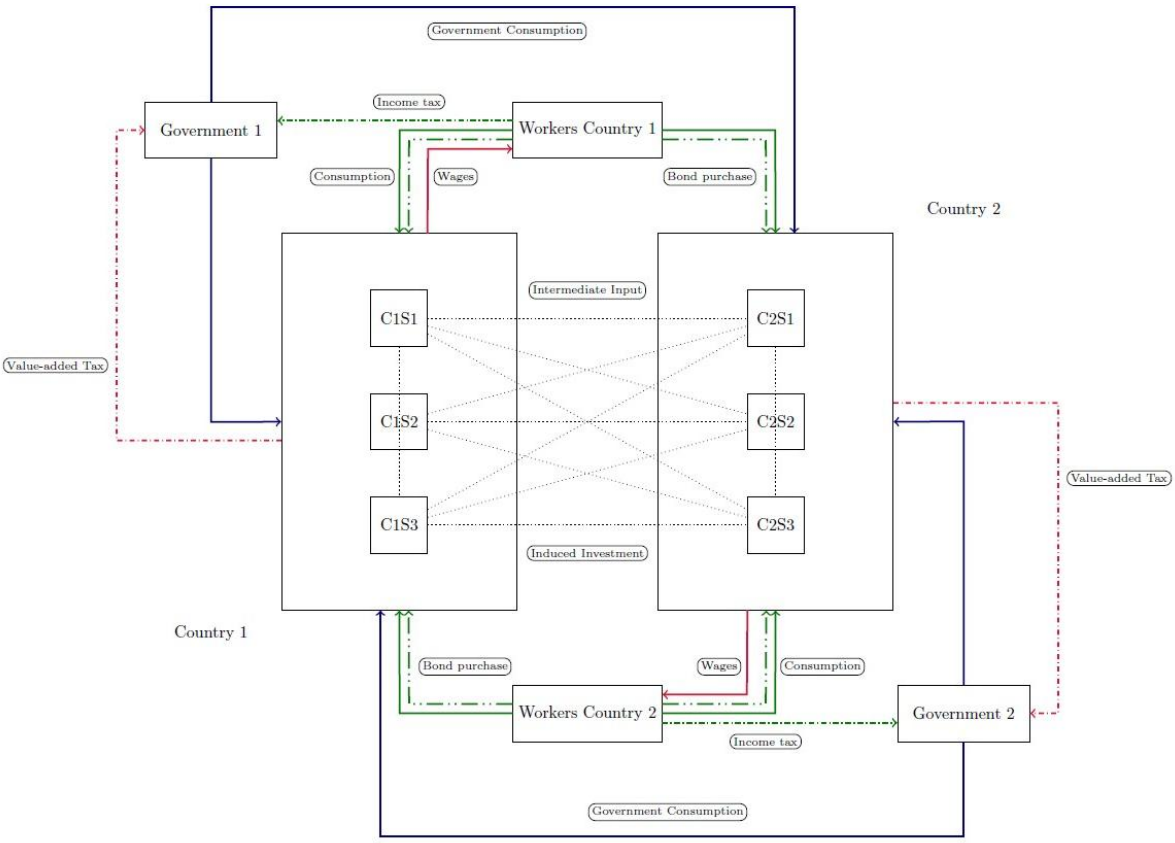



**Diagram 3. Model overview**

*The markups*

Markups are updated in response to the debt-to-capital ratios of firms in the country-sector, in line with Nomaler et al. (2025). This is where we use the capital stock measure $\kappa^{val}$. The equation for updating of the markups is as follows:

$$m_{i,t} = m_{i,t-1}\left[1 + \varpi\left(\frac{B_{t-1}^i}{\kappa_{i,t-1}^{val}} - \bar{d}_i\right)\right]$$

where, for each country-sector $i$, $m_i$ is (as introduced earlier) is the markup rate, $B^i_{t-1}$ is the accumulated debt stock as of time $t-1$, $\varpi$ and $\bar{d}_i$ are parameters, the latter indicating a universal target for the debt-capital ratio. We will refer to the debt-capital ratio $B^i_{t-1}/\kappa^{val}_{i,t-1}$ as the leverage ratio.

*Autonomous consumption (consumption smoothing)*

It is important that autonomous consumption is not exogenous, although it is independent of current income (or GDP). Autonomous spending in nominal terms (matrix **Φ**) depends on households' assets (wealth). For this, we introduce a new variable $\hat{\mathbf{Z}}$, which is an $n_{cs} \times n_c$ diagonal matrix that contains on its diagonal elements the total nominal autonomous consumption expenditures by all households in the column country. We then assume

$$\hat{\mathbf{Z}} = \hat{\boldsymbol{\zeta}}\hat{\boldsymbol{\Omega}}$$

where $\hat{\boldsymbol{\Omega}}$ ($n_c \times n_c$) is a diagonal matrix with the stocks of financial wealth of households (i.e., the bond holdings of households) on the main diagonal, and $\hat{\boldsymbol{\zeta}}$ (also $n_c \times n_c$) has the marginal propensity to spend out of wealth (specific to each country) on the main diagonal. Furthermore, we assume that the distribution of total autonomous consumption over the supplying country-sectors follows the same pattern as for induced consumption, which implies that the matrix **R** can be used to construct matrix **Φ**:

$$\boldsymbol{\Phi} = \mathbf{R}\hat{\mathbf{Z}}$$

As in Nomaler, Spinola and Verspagen (2021) we assume that the marginal propensities to spend out of wealth ($\hat{\boldsymbol{\zeta}}$) are endogenous themselves and follow a logic of consumption smoothing:

$$\zeta_{t,i} = \zeta_{t-1,i}\left(1 + \iota\left(\bar{E}_i - E_{it-1,i}\right)\right)$$

where $E_{t,i}$ is the employment rate in country $i$, $\bar{E}_i$ is a "neutral" employment rate, and $\iota$ is an adjustment parameter. This equation says that if employment is below the rate that is considered to be neutral by households, induced consumption spending will be supplemented by an increase of spending from accumulated savings (financial wealth). On the other hand, if the employment rate is higher than the neutral rate, households will increase "savings for a rainy day" by decreasing the marginal propensity to spend out of wealth. This consumption smoothing mechanism works as a stabilizer, as is shown by the elaborate stability analysis in Nomaler, Spinola and Verspagen (2021).

Compared to the earlier paper, we introduce one additional mechanism that affects consumption smoothing, which is to endogenize the neutral rate of employment $\bar{E}$. Whereas this was completely exogenous in Nomaler, Spinola and Verspagen (2021), here we allow this "parameter" to adjust on the basis of historical experience

$$\bar{E}_{i,t} = \epsilon\tilde{E}_{i,t-1} + (1-\epsilon)\bar{\bar{E}}_i$$

where $\tilde{E}_{i,t}$ is the average of the employment rate over a time span from $t$ to a fixed number of periods ago (we use 50 periods in the simulations below), $\bar{\bar{E}}_i$ is a parameter (the "truly" neutral employment rate), and $\epsilon$ in $[0..1]$ is a weight parameter. Thus, the neutral employment rate is a weighted average between the historical employment rate and an exogenous value.

*Wages*

We assume that the exchange rate between countries in the model is fixed, i.e., that global economy as described in the model is a monetary union. This rules out any exchange rate adjustment in response to the building up of foreign debt. In order to put a (potential) limit on foreign debt, we introduce a mechanism in which wages are adjusted in response to foreign debt:

$$w_{i,t} = w_{i,t-1}\left(1 - \chi\left[\Xi_{i,t-1} - \tilde{\Xi}_{i,t-1}\right]\right)$$

where $\mathbf{w}_i$ is the $n_s \times 1$ part of the wage vector $\mathbf{w}$ that refers to country $i$, $\chi$ is a parameter in $[0..1]$, $\Xi_{i,t}$ is the ratio of foreign debt to GDP in the country in period $t$, and $\tilde{\Xi}_{i,t}$ is the average foreign debt to GDP ratio over a time span from $t$ to a fixed number of periods ago. Thus, if foreign debt rises above (below) its historical average, wages will fall (rise), leading to the country becoming more (less) competitive (through the various Armington price elasticities) and spending less (more) out of wages. This is expected to raise (decrease) the current account and hence bring foreign debt down (up).

*Other dynamic variables*

There are other dynamic variables that should be endogenized in the future full version of the model, but which we have left as purely exogenous in this conceptual version. This includes foremost the dynamics of the Armington share parameters $\alpha$, $\beta$ and $\gamma$ and the closely related matrices $\mathbf{N}^{basesect}$, $\mathbf{R}^{sect}$ and $\mathbf{B}^{sect}$. These are important variables that, together, match global demand and supply and determine bilateral trade flows. These variables can be expected to vary over time based on technological change (in particular product innovation), as well as on policy-related trends such as trade liberalization (or restrictions), globalization, ageing, etc.

It also remains to be determined how the green and digital technological revolution is incorporated into the model. For the green dimension, a link with greenhouse gas emissions can easily be built into the model through existing satellite accounts data on emissions. Digital technological change may have its largest impact on labour productivity and labour market participation.

## 5. Simulations of the dynamic model

The complete model now consists of a short-run part that solves for activity levels (output and employment), and bilateral trade, and a set of intertemporal updating mechanisms that connect the short-run solutions by changing the variables that the short-run model takes as given. One way to analyse this model is by simulation: initializing the model with a set of coherent values for the variables and the parameters, and then simply calculate a sequence of short run solutions and dynamic updating that connects them. This section presents simulations like this for a two-country-three-sector version of the model. These simulations are intended as examples of the dynamics that are generated by the model. A full stability analysis based on searching parameter space in a systematic manner is beyond the scope of this conceptual model.

The base simulation run is initialized as described in the Appendix (parameter values and initial values). In addition, no government is implemented in the simulations, which implies that the value added tax rate is zero. The simulation includes all short-run and dynamic equations as explained so far. However, rather than running the equations as they are, we introduce a smoothing mechanism, which is aimed at reducing short-run fluctuations of the results. This is

implemented on a selection of variables. When implemented, it means that we calculate the new value of the variable based on the equations specified above, but instead of the new value of this variable being equal to this calculated value, it is set to the average of a fixed number of most recent historical values and the newly calculated value.

This smoothing procedure is applied to the markups $m$ (equal to the average of 3 historical values plus the newly calculated value), all entries in $\mathbf{N}^{base}$ and $\mathbf{B}^{base}$ (both using 5 historical values plus the newly calculated value), both values of $\zeta$ (19 historical values plus the newly calculated value), the wage rates $\mathbf{w}$ (10 historical values plus the newly calculated value), autonomous consumption $\mathbf{\Phi}$ (5 historical values plus the newly calculated value), and the $\mathbf{K}$ matrix (5 historical values plus the newly calculated value).

The results for a number of key variables of the base simulation run are shown in Figure 2. As there is no productivity growth, this is a stationary global economy. In this case, there seems to be a stationary stable state, in which employment rates are at (very close to) the neutral value (7%), foreign debt as a percentage of GDP is also stable, and the propensity to consume out of financial wealth is stable but slightly different between the two countries. This stationary stable state is reached after initial dampening fluctuations. It takes almost the entire length of the simulation (2000 periods) to come close to the stationary stable state.

Revealed comparative advantages (calculated on total exports) are relatively stable over the entire run. We calculate these in the usual way but scale them between – 1 and 1. Country 1 has a clear comparative advantage in Sector 1, and Country 2 has a clear comparative advantage in Sector 3. Comparative advantages are very weak (i.e., very close to zero) in Sector 3. Connected to this specialization structure, country 1 (2) runs the current account deficit (surplus), resulting in a long-run stable debt-to-GDP ratio that is positive (negative) in Country 2 (1).




**Figure 2. Simulation results for the base run**

In a next simulation (Figure 3), we decrease the value of all Armington elasticities to 1. Somewhat counter-intuitively, the lesser sensitivity of trade to process leads to more pronounced fluctuations in the later parts of the simulation run. In this case, the unemployment rates and the

debt-to-GDP ratios never flatten out within the time span of the simulation. Nevertheless, the simulation still shows convergence to a stable stationary state that is qualitatively similar to the base run.




**Figure 3. Simulation results for Armington elasticities = 1**

We conclude this section with a simulation which productivity growth is introduced onwards from period 150. To make things a little more interesting, we choose to implement differential rates of growth between the country-sectors: Sector 2 in Country 1 and Sector 1 in Country 2 (the sectors in which the countries have a clear comparative disadvantage) have fastest productivity growth, at 0.25% per period, in Sector 1 in Country 1 and Sector 3 in Country 2 productivity grows at a slightly lower rate of 0.22% period and, and in Sector 3 in Country 1 and Sector 2 in Country 2, productivity does not grow at all. Although these productivity growth rate differentials will show some interesting results, the exercise of varying productivity growth rates alone must be considered a preliminary exercise, because also the $\alpha$, $\beta$, and $\gamma$ values should change to avoid a situation in which countries with a comparatively low productivity level serve a relatively large part of the international market.

In this run, wages rates are increased in parallel with productivity in order to avoid severe price deflation (the simulation runs fine without this). We also keep the value of all Armington elasticities at 1, so as to avoid heavy fluctuations.

The results of the last simulation are in Figure 4. We do not document trends for GDP, which is growing in line with productivity, but stay with the four variables documented so far. This shows that we still have a stable state, in which unemployment fluctuates around the neutral value of 7%. However, propensities to consume out of financial wealth converge to lower values than before.

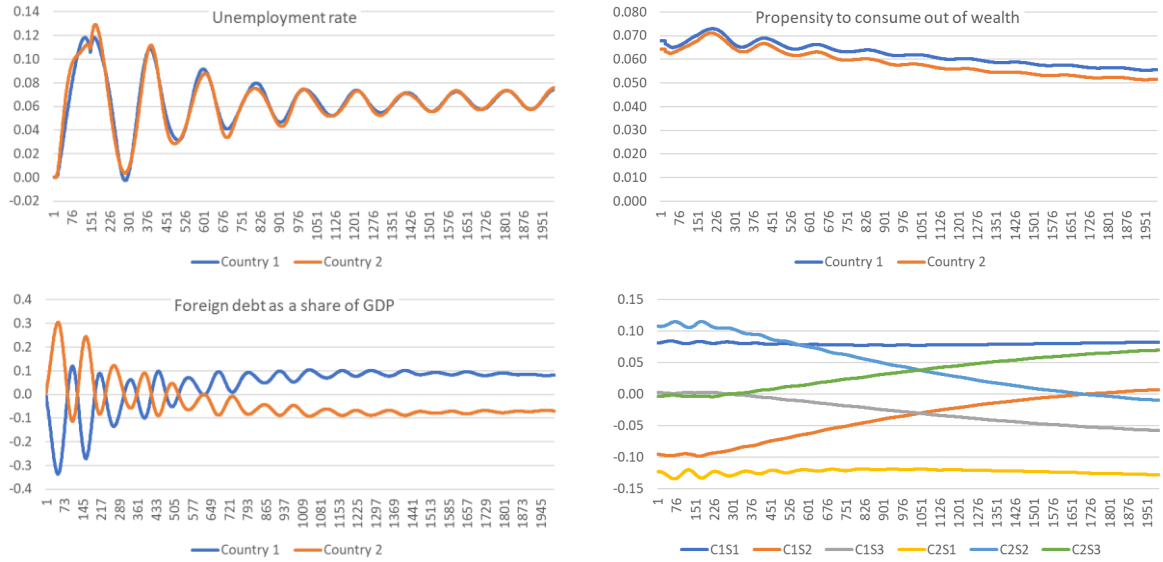



**Figure 4. Simulation results for Armington elasticities = 1 and divergent productivity growth rates**

Most importantly, the diverging productivity growth rates lead to a dynamic change in specialization patterns. Whereas in the base run, Sector 3 showed almost no specialization, now Country 2 gradually becomes specialized in this Sector, while at the same time Country 2 looses its RCA in Sector 2. These changes are gradual, but they do lead to a reversal of the international debt positions: now Country 1 (2) becomes the country with a surplus (debt).

## 6. Numerical calculation of the steady state

The simulation results in Figures 2 and 3 in the previous section suggest that there is a steady state and that the specific implementations (starting values and parameters) that we used makes the model converge to this steady state. We propose that this steady state can be used to analyse the model. In the case of Figures 2 and 3, where there is no technological change or productivity growth, the steady state captures the long-run tendency of the model. While we do not have productivity growth in this conceptual version of the model (other than in the simulations of Figure 4), our proposal about the use and interpretation of this steady state extends to future versions of the model that will include technological progress and other dynamic processes.

In this case, we still refer to a steady state without productivity growth, i.e., with static long-run conditions, leading to different steady states over time. To see this, we must realize that for cases as in Figures 2 and 3, only one steady state can be calculated, because this steady state is independent of initial conditions and long-run conditions will be constant in the steady state.[10] But if productivity changes over time, as it does in Figure 4, each individual period has its own steady state, dependent on productivity levels in that period. Graphing those subsequent steady states as a time series produces a tool to analyse the outcomes of the model. As already mentioned in section 2 above, this is an approach that is reminiscent of the procedure used in CGE models.

[10] Because we have not given any proof of the uniqueness and stability of the steady state, we must formally allow for multiple steady states to exist, and/or for steady states not to reachable from (some) initial values. However, simulations suggest that the steady state is unique and stable under a wide range of parameter settings.

We will now conceptualize and define this steady state, thus enabling us to solve the model numerically for the steady state. As will be shown, the definition of the steady state depends to a very large extent on the stock-flow consistent nature of the model, as it requires us, among other things, to solve for a solution where financial flows will be zero.

The derivation of this steady state starts from the observation that prices **p** will be constant as all other variables. This implies that the replacement value capital stock $\kappa_{i,t}^{val}$ and the capital stock $\kappa_{i,t-1}^{pc}$ are related only by prices and the sectoral composition of the capital stock: the sum of columns in $\hat{\mathbf{p}}\mathbf{N}^{base}$ is the price of a unit of productive capacity in column in the steady state. We will denote this price by $p_i^{pc}$. Multiplying any $\kappa_i^{pc}$ with its respective price will yield the steady state value of the replacement value $\kappa_i^{val}$.

Next, we look at the equations for the change of the investment propensity $h$. In the steady state, this must be constant which implies[11]

$$h_{i,t} - h_{i,t-1} = h_{i,t-1}\xi\left(\frac{Q_{t-1,i}}{\kappa_{i,t-1}^{pc}} - \bar{u}_i\right) = 0 \rightarrow \kappa_i^{pc} = \frac{Q_i}{\bar{u}_i}$$

This leads to

$$\kappa_i^{val} = \kappa_i^{pc} p_i^{pc} = \frac{Q_i}{\bar{u}_i} p_i^{pc}$$

Next, we note that in the steady state the markup $m$ must also be constant. Because the change of the markup depends on the deviation of the leverage ratio from its neutral value, a constant markup requires that the leverage ratio is equal to its neutral value:

$$m_{i,t} - m_{i,t-1} = m_{i,t-1}\varpi\left(\frac{D_{t-1}^i}{\kappa_{i,t-1}^{val}} - \bar{d}_i\right) = 0 \rightarrow D_i = \bar{d}_i \kappa_i^{val} = \bar{d}_i p_i^{pc} \frac{Q_i}{\bar{u}_i}$$

This equation gives us the steady state debt stocks of the firms in each country-sector as a function of its steady state capital stock and steady state prices, and the steady state capital stock, in turn is determined by steady state output.

Now it is time to take advantage of the stock-flow consistent nature of the model (as represented in the transactions matrix and balance sheets), which tells us that the global sum of outstanding debt by all firms in all country-sectors is equal to the sum of financial wealth of all households globally.[12] The last equation thus implies that in the steady state, global debt ($D$) or financial wealth depends only on output levels, prices, and a set of parameters and exogenous variables.

This brings us back to Section 4.4, and Diagram 2. In order to solve what we have called the short-run model, i.e., to solve for the levels of output and employment (in this case the steady state levels of those variables) in each country-sector, we need both the (inverse) multiplier matrix, and autonomous demand. Because we abstract from a role for the government, autonomous demand is only private autonomous demand, which is determined as the multiplication of the propensity to spend out financial wealth, $\zeta$, the respective stocks of financial wealth, and the matrix **R** that distributes total autonomous demand over the country-sectors.

As shown above, any level of steady state gross output **Q** as determined by the multiplier matrix and autonomous demand leads to specific values of the stock of global wealth. A first condition for

[11] We ignore the trivial option $h = 0$ and, below, $m = 0$.

[12] In a model with government, government debt would also have to be included in the model. We abstract from this now.

deriving the steady state is that the stock of global financial wealth and its distribution to households in the various countries yields a vector of autonomous demand that, when multiplied with the multiplier matrix, is consistent with the same outcome values of global wealth.

Diagram 2 also shows that in order to solve for output and employment, we need values of the wage vector **w**, the markup vector **m** and the propensities to invest **h** (these are all endogenous variables), as well as various other parameters or exogenous variables (such as the three types of Armington share parameters and Armington elasticities). For **h** we can derive steady state values from the equation for the evolution of the capital stock: $\kappa_{t,i}^{pc} = (1-\delta_i)\kappa_{t-1,i}^{pc} + h_{i,t}Q_{i,t}$. A constant value of $\kappa_i^{pc}$ implies

$$\kappa_{t,i}^{pc} - \kappa_{t-1,i}^{pc} = h_{i,t}Q_{i,t} - \delta_i\kappa_{t-1,i}^{pc} = 0 \rightarrow h_i = \delta_i\frac{\kappa_i^{pc}}{Q_i} = \frac{\delta_i}{\bar{u}_i}$$

In our proposal for the procedure to solve for the steady state, and with **h** fixed to its steady state values, the endogenous variables **w**, **m**, **p** become decision variables. We will let **m** and **p** vary freely (in the positive domain), but for **w** we follow a slightly more restrictive approach.

Remember that in the specification of the model so far, initial wage rates were exogenous, but we also specified a dynamic adjustment mechanism. The purpose of this mechanism was to avoid excessive international debt, and we specified that countries who run into large international debt levels (relative to their GDP), would see a downward adjustment of their wages. In the optimization procedure for finding a steady state, we do not parametrize this mechanism as we did in the simulations. Instead, we use a set of exogenous wages rates (e.g., similar to those used as initial values in the simulations) and then introduce a set of decision variables to mimic the adjustment mechanism. Specifically, we introduce a (positive) adjustment parameter, denoted by $\lambda_i$ for each country that we use to multiply the exogenous wage rates, and the result is used in the calculations (e.g., for the calculation of prices). For one of the countries, this adjustment parameter is fixed to 1, and for all other countries, it is a decision variable.

As an additional set of decision variables, we use the debt levels of all country-sectors, the values of $\zeta$ for each country, and a set of distribution parameters $\omega_i$ that represent the share of financial global wealth that households in country $i$ hold (as the $\omega$s have to sum to 1, there are $1-nc$ such parameters). By manipulating these decision variables, we will generate a short-run solution for the model, and the goal of our numerical optimization procedure is to yield a short-run solution (activity levels) that is consistent with the steady state assumptions. This is operationalized a constrained optimization problem, where the constraints (rather than the goal function) carry the burden of finding the steady state.

As a first restriction, the short-run solution that results from the current values of the decision variables will have to yield debt levels of the country-sectors that are equal to the values that we used as inputs, i.e., the decision variables. Another restrictions that needs to be satisfied for the short-run outcome to be a steady state results from the equation for the dynamic change of $\zeta$. For $\zeta$ to remain constant, we need[13]

$$\zeta_{t,i} - \zeta_{t-1,i} = \iota\zeta_{t-1,i}\left(\bar{E}_i - E_{it-1,i}\right) \rightarrow E_i = \bar{E}_i$$

which says that the employment rate needs to be at its neutral value. In this case we will take $\bar{E}_i$ as a parameter. We will also assume that the labour force is exogenous and fixed, so that the

[13] Once again, we ignore the trivial solution $\zeta = 0$.

employment rates $E_i$ can readily be calculated from the employment levels that result from the short run solution of the model.

Finally, we need a restriction that says that the change of the debt stocks and financial wealth of the households in each country does not change. Any change of these variables implies that we are not in a steady state, because autonomous will change. This is another consequence of the stock-flow consistent nature of the model. In this case it means that the cash flows of households (in all countries separately) and of firms (in all country-sectors separately) need to be zero. Any household cash flow that is non-zero would imply that the stock of financial wealth of those households would change, which would violate the steady conditions because it would induce a change in autonomous spending. Similarly, a non-zero cash flow of firms in any country-sector would induce a change in debt of that country-sector, and, because all debt needs to be held by households, a change in the financial wealth stock of households, leading, again to a change in autonomous spending.

For households, this cash flow restriction means that "induced savings" (the difference between disposable income and induced consumption spending) must be equal to autonomous spending (which results endogenously in the optimization procedure). For firms, it means that operational profits (i.e., the difference between turnover and variable costs) must be equal to investment. Both these restrictions are formulated in nominal terms, because the financial flows are also denoted nominally.

The numerical procedure that finds the steady state is summarized in Diagram 4.

| |
|---|
| **Step 1**: initialize the decision variables: prices ($p$), markups ($m$), propensities to spend out of wealth ($\zeta$), debt stocks of firms in country-sectors ($D$), wealth distribution parameters ($\omega$) and relative wage factors ($\lambda$); then proceed to **step 2** |
| **Step 2**: calculate the short-run solution of the model using decision variables, parameters and exogenous variables; calculate debt stocks for all country-sectors under the assumption that this short-run solution is a steady state; then proceed to **step 3** |
| **Step 3**: evaluate all constraints that result from the steady state definition:<br>1. Is the sum of squared deviations between prices as decision variables and prices as outcomes close enough to 0?<br>2. Is the sum of squared deviations between firm debt stocks as decision variables and firm debt stocks as outcomes close enough to 0?<br>3. Is the sum of squared deviations between employment rates as outcomes and the neutral employment rates (parameters) close enough to 0?<br>4. Is the sum of squared deviations between the cash flow of households (induced savings minus autonomous spending) close enough to 0?<br>5. Is the sum of squared deviations between the cash flow of firms (operational profits minus investment) close enough to 0?<br>(Note that all constraints together define the steady state; it is optional to define a goal function, e.g., the sum of squared deviations as in constraint 2, that is to be minimized: if constraint 2 holds, the goal function will be minimized); then proceed to **step 4** |
| **Step 4**: if all of the constraints from **step 3** are satisfied, the steady state has been found. In this case the algorithm is ended. If not all constraints are satisfied, proceed to **step 5** |
| **Step 5**: adjust decision variables (based on numerically calculated derivatives) and return to **step 2** |

**Diagram 4. Numerical algorithm to find the steady state**

### 6.1. Steady states of the 2-countries-3-sectors model

We solved for the steady state using the optimization approach that is outlined in Diagram 4. The results are documented in Table 4. These optimizations are run for the same conditions as used in the base simulation run of the previous section. Programs 1 and 2 were run with different starting values for the firm debt levels as decision variables. These two programs yield partially different values of the decision variables, which indicates there is no unique global steady state for this setting of the model.

The third column shows the difference between Programs 1 and 2, and this shows that the propensity to spend out of wealth ($\zeta$) and the share of the two countries of total holdings of global financial wealth differ between the programs (both of these are decision variables). As a result, foreign assets of the two countries differ. In Program 1, country 1 becomes indebted to country 2, but in Program 2 it is the other way around.

**Table 4. Outcomes of optimization programs**

| | | *Program 1* | *Program 2* | *Difference* | *Program 3* |
|---|---|---|---|---|---|
| *Decision variables* | | | | | |
| *Prices* | **C1S1** | 1.00037 | 1.00037 | 0.00000 | 1.00037 |
| | **C1S2** | 1.00040 | 1.00040 | 0.00000 | 1.00040 |
| | **C1S3** | 1.00034 | 1.00034 | 0.00000 | 1.00034 |
| | **C2S1** | 1.00122 | 1.00122 | 0.00000 | 1.00122 |
| | **C2S2** | 1.00119 | 1.00119 | 0.00000 | 1.00119 |
| | **C2S3** | 1.00117 | 1.00117 | 0.00000 | 1.00117 |
| *Wage adjustment (=1 for C1)* | **C2** | 1.00153 | 1.00153 | 0.00000 | 1.00153 |
| *markups* | **C1S1** | 0.20004 | 0.20004 | 0.00000 | 0.20004 |
| | **C1S2** | 0.20004 | 0.20004 | 0.00000 | 0.20004 |
| | **C1S3** | 0.20005 | 0.20005 | 0.00000 | 0.20005 |
| | **C2S1** | 0.19996 | 0.19996 | 0.00000 | 0.19996 |
| | **C2S2** | 0.19996 | 0.19996 | 0.00000 | 0.19996 |
| | **C2S3** | 0.19997 | 0.19997 | 0.00000 | 0.19997 |
| *Propensity to spend out of wealth ($\zeta$)* | **C1** | 0.07171 | 0.06452 | -0.00719 | 0.06591 |
| | **C2** | 0.06155 | 0.06756 | 0.00601 | 0.06620 |
| *Firm debt levels* | **C1S1** | 5419.6 | 5419.6 | 0.00000 | 5419.6 |
| | **C1S2** | 8461.9 | 8461.9 | 0.00000 | 8461.9 |
| | **C1S3** | 6959.9 | 6959.9 | 0.00000 | 6959.9 |
| | **C2S1** | 6307.0 | 6307.0 | 0.00000 | 6307.0 |
| | **C2S2** | 8607.4 | 8607.4 | 0.00000 | 8607.4 |
| | **C2S3** | 7418.6 | 7418.6 | 0.00000 | 7418.6 |
| *Share of holdings of global debt (=1 for C1)* | **C2** | 0.55633 | 0.50688 | -0.04945 | 0.517 |
| *Selected outcome variables* | | | | | |
| *Unemployment rate* | **C1** | 0.07000 | 0.07000 | 0.00000 | 0.07000 |
| | **C2** | 0.07000 | 0.07000 | 0.00000 | 0.07000 |
| *Foreign assets (– indicates debt)* | **C1** | -1686.4 | 448.7 | 2135.1 | 0.00 |
| | **C2** | 1686.4 | -448.7 | -2135.1 | 0.00 |

Note: in all cases, the value of the objective function converged to 0 (within tolerance limits)

The indeterminacy states that with the constraints employed in the optimization so far, the total level of global financial wealth (or its counterpart, debt) is uniquely determined, but the distribution of these holdings between the two countries is not determined. This distribution determines financial wealth of households in each of the countries, and this in turn fixes autonomous demand by these households. Because autonomous demand in each country is unique in the steady state, the other side of the indeterminacy is that the steady state levels of $\zeta$ need to adjust to keep autonomous demand constant. For example, in Program 1, country 1 holds a lower share of global financial wealth than in Program 2, and hence $\zeta$ is higher in country 1 in Program 1 than in Program 2.

This indeterminacy can be eliminated by specifying the additional restriction that no country can hold any foreign assets (negative or positive). In terms of Diagram 4, we need to add this additional constraint in **step 3**:

6. Is the sum (over countries) of squared deviations between the wealth stock of households and the sum of debt stocks of all firms in each country close enough to 0?

The algorithm with this additional constraint is implemented in Program 3, which indeed results in zero debt of the two countries (final row of Table 4). Effectively, without this last restriction, we have an infinite number of combinations between, on the one hand, the $\zeta$s, and on the other hand the distribution of global financial wealth holdings. The restriction on foreign assets fixes this to specific combination of these decision variables that yields zero foreign assets.

### 6.2. Steady state of the 6-countries-14-sectors empirical model with government

As a final exercise, we now proceed to derive the steady state for the real-world 2019 $6 \times 14$ model. As in the Python Box in section 4, we have to deal with the role of the government in this model. Conceptually, this means we need to introduce the possibility of government debt and income taxes as in the variable $inctax_i$ that was introduced in the Python Box in section 4 (we multiply $c_i$ with $(1 - inctax_i)$. We consider the procedure for dealing with the impact of government that we present and use as preliminary. This needs to be developed further in a future version of the model, so we present this model only as a proof of concept.

Theoretically, the steady state requires that each government runs a balanced budget. If this would not be the case, government debt would change (diminish if the government runs a surplus, increase if it runs a deficit, and stock-flow consistency then requires that debt and/or wealth stocks of the other agents (firms and household) will also change. This violates the no-change definition of the steady state. Thus, we need to add another condition in **step 3** in Diagram 4 (we also keep the sixth condition introduced in the text above):

7. Is the sum (over countries) of squares of the differences of nominal value of government revenues (sum of value added and income tax revenues) minus nominal government (autonomous) expenditures for all governments sufficiently close to 0?

While this condition must be undisputed, there is a choice in treating the new government-related variables either as exogenous or as decision variables. If the government budget must be balanced, both tax rates as well as expenditure levels may be used to achieve this, so all these variables are

candidates for being decision variables.[14] However, having too many decision variables will create a tendency to introduce indeterminacies in the steady state. For example, if both taxes and expenditures are decision variables, a balanced government budget may be achieved with high taxes and high expenditures, or with low taxes and low expenditures.

As a preliminary setup, we make both value added and income tax rates exogenous and use government expenditures in real terms and per sector as decision variables. We implement the restriction that governments will not import anything, i.e., all autonomous government expenditures are spent exclusively in the domestic economy. This introduces 84 extra decision variables (6 countries × 14 potential sectors to spend on) in the algorithm as in Diagram 4 (which also has the extra conditions 6 and 7).

In this setup, there is no special consideration of government debt. As the government can be seen as one of the 14 sectors (see Table 1, sector O-Q), government debt exists in the model, but it is treated in exactly the same way as private debt, i.e., by using the neutral leverage ratio). In reality, government debt is different from private debt, and we could adjust the model accordingly, i.e., by setting government debt in the steady state to a fixed fraction of GDP. This is left for future development of the model.

By implementing this setup in the $2 \times 3$ theoretical model, we discovered that there is still a degree of indeterminacy in this model. This is related to the choice of which sectors the government targets with its autonomous expenditures (which are decision variables). In the small model, there are $2 \times 3 = 6$ extra decision variables related to government expenditures. With this, we could find one steady state in which the government targeted most of its expenditures on sector 1, and another steady state in which it targeted sector 2. This means that there is still some indeterminacy in this setup with 6 government decision variables.

At this stage, we have not yet been able to investigate this indeterminacy completely, hence we proceeded with the steady state calculation algorithm as explained so far (Diagram 4 with extra conditions 6 and 7 and 84 extra decision variables related to government spending). We leave further investigating of this issue to a future version of the model. The challenge here is to find a setup with less decision variables, so that we have exactly enough degrees of freedom to find a unique steady state.

Our implementation of the optimization algorithm for finding the steady state builds on a generic solver in Matlab (the *fmincon* function for constrained minimization).[15] We were able to find a steady state using this algorithm for the $6 \times 14$ real-world data model for 2019, but this computation takes a long time on a high-end Windows 11 desktop computer. Therefore, we changed some of the parameters to values that seemed to lead to faster convergence in the early stage of the optimization. In particular, we set the depreciation rate of capital to 0.1 (compare a value of 0.05 in the simulations of section 5 and optimizations of the small-scale model), all value added tax rates to 0.05 (note that these include subsidies, so in reality many of these rates are negative), the income tax rate in China to 0.075 (compare the value of 0.0307 in the Python Box in Section 4), the propensity to consume out of wages in China to 0.55 (compare to the value 0.62 in the Python Box), and the propensity to consume out of wages in the UK to 0.65 (compare to the value 0.87 in the Python Box).

[14] We have also experimented with versions of the model where the government pays unemployment benefits out of its budget. We leave further development of this option to future versions of the model and hence disregard unemployment benefits in this paper.

[15] The Matlab script has been translated to Python using AI, but to date this Python version remains untested. We will make it available after further testing.

Finally, we also need data on labour markets in the six countries. Data on wages are already included in the MRIO table, but we also need values for the labour force in each country and for the neutral (un)employment rate. For this, we used data from the ILO ("modelled ILO estimates for the unemployment rate") as found in World Development Indicators, and data on labour force from the World development Indicators). $\bar{E}_i$ was calculated as the average of the actual employment rate over 2017-19, and the labour force in terms of the number of working hours was calculated by applying the employment rate for 2019 to the 2019 actual worked hours from Exiobase. This leads to the data in Table 5 that were used in the steady state calculation.

**Table 5. Labour market data used in the calculation of the steady state**

| | $1-\bar{E}_i$ | labour force (number of working hours) |
|---|---|---|
| **EU27** | 0.0736 | 521,575 |
| **CHN** | 0.0445 | 2,427,068 |
| **GBR** | 0.0406 | 82,344 |
| **JPN** | 0.0255 | 177,071 |
| **USA** | 0.0397 | 460,425 |
| **ROW** | 0.0632 | 9,020,616 |

Note that all these parameters have only been calibrated in a very loose sense. Calibration needs much more attention in a future version of the model.

The steady state that we derived likely suffers from indeterminacy with regard to which sectors are targeted with government expenditures, which makes the steady state results at the sectoral level less interesting. Table 6 reports the values of three decision variables at the country level, and which are also of crucial interest for a characterization of the distribution between global world regions. We see that EU27 captures a 20% share of global financial wealth, while in the USA this is slightly higher at 22.5%. China follows at 17% and Japan and the UK, as individual smaller countries are much below this. Almost one third of global wealth is attributed to ROW. Note that these percentages apply both to wealth holdings and to debts, because we have ruled out any international debt as part of the steady state definition.

The relative wage variable is set to 1 for EU27, and the values for other countries are a multiplicative factor that is applied to actual 2019 wage rates. Thus, we see that Chinese wages are more than 11% lower in the steady state and Japanese wages are about 8% lower. On the other hand, US and UK wages are slightly higher in the steady state.

**Table 6. Steady state outcomes for the real-world model, 2019**

| | Share of global wealth ($\omega$) | Relative wages ($\lambda$) | Propensity to spend out of wealth ($\zeta$) |
|---|---|---|---|
| **EU27** | 0.203 | 1.000 | 0.137 |
| **CHN** | 0.173 | 0.888 | 0.133 |
| **GBR** | 0.030 | 1.056 | 0.159 |
| **JPN** | 0.070 | 0.919 | 0.176 |
| **USA** | 0.225 | 1.031 | 0.204 |
| **ROW** | 0.299 | 0.982 | 0.167 |

Finally, the propensity to spend (autonomously) out of wealth varies relatively little between countries. This is highest in the USA at slightly over 20%, and lowest in China at slightly over 13%. EU27 also ranks low with a value just a little bit higher than China.

These steady state values must be taken as a proof of concept, rather than realistic estimates. The fact that we were able to find these values, and that they look to be reasonable, is encouraging for the prospects of further modelling.

## 7. Summary of the conceptual model and next steps

The conceptual model that we presented in this paper is built on the principles of dynamic input-output modelling, stock-flow consistency, and the supermultiplier. It provides a way to model the global economy with a strong emphasis on structural change. The model consists of the solution of a short-run model that takes long-run variables such as productivity and competitiveness (represented by the Armington share parameters) as given, and a set of updating rules for the long-run variables that connect the short-run states. Although do provide updating rules for some of the long-run variables, we leave productivity and other long-run variables unspecified in this conceptual version of the model.

We are able to numerically calculate a steady state of a real-world model with six countries or country-blocks and 14 sectors. The steady state takes productivity as fixed and exogenous. We propose that the calculation of these steady states for a sequence of yearly models in which productivity and technology more generally changes over time is the way to develop this model further.

In such a future version of the model, we suggest taking the following steps:

1. Integrate (semi-)endogenous green and digital technological change and the impact that it has on the global economy into the model. This will likely also require the explicit inclusion of climate and/or environmental processes into the model, because this is where green technology will have its main desired impact. This can be done, for example, by focusing on greenhouse gas emissions and climate change by including emissions into the MRIO that underlies the model. In this way, the impact of green technology on emissions can be included in the model without much effort. Digital technological change will likely have an impact on productivity and on green innovation, which are processes that also will have to be included into the model. Product innovation is also a form of innovation that needs to be included into the model, for example in the form of its impact on competitiveness and the Armington shares parameters.
2. A decision needs to be made with regard to the exact empirical setting in which the model will be developed. Currently, we used six countries and 14 sectors, but this may not be ideal. We may, for example, use individual European countries rather than EU27, or use less trading partners, and possibly a more detailed sectoral breakdown. This will have to be decided on the basis of demands from stakeholders as well as data availability.
3. The role of government needs to be specified in more detail, both in terms of the SFC content of the model (government debt and fiscal and monetary policy), and in terms of technology policy. This includes a role for public finance (how are government expenditures distributed over the economy?), social security (unemployment benefits and other forms of social security), and industrial and innovation policy. Related to this, we need to provide a better way of identifying the exact number of degrees of freedom that are necessary to derive a unique steady state, to avoid the indeterminacies in the steady state that we were able to derive so far.

4. The SFC part of the model can possibly be further enhanced by incorporating elements of the existing SFC modelling tradition, especially with regard to monetary and fiscal policy and financial markets. This includes the use of detailed statistics on financial stocks and flows by economic agents, which are available for many countries. Currently, we have used this kind of data in the calculation of the steady state, but we relied only on data on stocks and flows that we are able to derive from the MRIO data and a number of additional assumptions. Including more realistic data may also enhance the sophistication of the steady state calculations.

**List of notation (parameters and variables)**

| Symbol | Explanation |
|---|---|
| $p_i$ | Price of output delivered by country-sector $i$ (element of $\mathbf{p}$) |
| $\mathbf{p}$ | Column vector of prices of the country-sectors (with elements $p_i$) |
| $\rho_{kj}$ | Armington price elasticity for intermediate goods delivered by country-sector $k$ to country-sector $j$ |
| $b_{kj}^{sect}$ | Sectoral input coefficient: use of inputs produced by sector $k$ (in any country) and used in country-sector $j$, in real terms (element of matrix $\mathbf{B}^{sect}$) |
| $b_{ij}$ | Input coefficient: use of inputs produced by country-sector $i$ and used in country-sector $j$, in real terms (element of matrix $\mathbf{B}$) |
| $\alpha_{ij}$ | Armington share parameters for intermediate inputs ($\sum_z \alpha_{zj} = 1$) |
| $\mathbf{B}^{sect}$ | Matrix of input coefficients per sector, in real terms (with elements $b_{kj}^{sect}$) |
| $\mathbf{B}$ | Matrix of input coefficients, in real terms (with elements $b_{ij}$) |
| $w_j$ | Wage rate paid by country-sector $j$ (element of $\mathbf{w}$) |
| $\mathbf{w}$ | Column vector of wage rates (with elements $w_j$); also used as diagonal matrix $\widehat{\mathbf{w}}$ |
| $g_j$ | Labour coefficient (inverse labour productivity) in country-sector $j$ (elements of $\widehat{\mathbf{G}}$) |
| $\widehat{\mathbf{G}}$ | Diagonal matrix with elements $g_j$ on the main diagonal |
| $\tau_j$ | Value added tax rate applied to country-sector $j$, includes subsidies |
| $\tau_j^*$ | Value added tax rate as a fraction of total production costs |
| $m_j$ | Markup over variable production costs (intermediates and labour) in country-sector $j$ |
| $\widehat{\widehat{\mathbf{m}}}$ | Diagonal matrix with $1 + m_j(1 + \tau_j)$ |
| $\widehat{\widehat{\widehat{\mathbf{m}}}}$ | Diagonal matrix with $(1 + \tau_j)(1 + m_j)$ on the main diagonal |
| $\mathbf{I}$ | Identity matrix |
| $\mathbf{L}$ | Column vector of employment in the country-sectors |
| $\mathbf{y}$ | Column vector of labour income earned in the country-sectors |
| $c_i$ | Marginal propensity to consume out of labour income by workers employed in country-sector $i$ (elements of diagonal matrix $\hat{\mathbf{c}}$) |
| $\hat{\mathbf{c}}$ | Diagonal matrix with marginal propensity to consume out of wage income ($c_i$) on the main diagonal |
| $r_{ij}$ | Share of country-sector $i$ deliveries in total induced consumption of workers employed in country-sector $j$, nominal terms (elements of matrix $\mathbf{R}$) |
| $\mathbf{R}$ | Matrix of consumption coefficients, nominal terms (with elements $r_{ij}$) |
| $r_{kj}^{sect}$ | Sectoral consumption coefficients: share of sector $k$ deliveries in total induced consumption of workers employed in country-sector $j$, nominal terms (elements of matrix $\mathbf{R}^{sect}$) |
| $\mathbf{R}^{sect}$ | Matrix of sectoral consumption coefficients, nominal terms (with elements $r_{kj}^{sect}$) |
| $\varrho_{kj}$ | Armington price elasticity for consumption goods delivered by country-sector $k$ to workers employed in country-sector $j$ |
| $\beta_{ij}$ | Armington share parameters for consumption goods ($\sum_z \beta_{zj} = 1$) |
| $c_{ij}$ | Consumption-income coefficients: share of total wage income spent on goods delivered by country-sector $i$ to workers employed in country-sector $j$, nominal terms (elements of matrix $\mathbf{C}$) |
| $\mathbf{C}$ | Matrix of consumption-income coefficients, nominal terms (with elements $c_{ij}$) |
| $\mathbf{K}$ | Matrix of consumption-income coefficients, real terms |
| $\mathbf{\Phi}$ | Matrix of autonomous demand by households, with rows representing all country-sectors and columns representing countries, nominal terms |
| $\mathbf{\Phi}^g$ | Matrix of autonomous demand by governments, with rows representing all country-sectors and columns representing countries, nominal terms |

| Symbol | Explanation |
|---|---|
| $\mathbf{1}$ | Column vector with only ones |
| $\mathbf{f}$ | Column vector of total autonomous demand, real terms |
| $n_{ij}^{base}$ | Capital composition coefficients: contribution of country-sector $i$ to a unit of productive capacity in country-sector $j$, real terms (elements of matrix $\mathbf{N}^{base}$) |
| $\mathbf{N}^{base}$ | Matrix of capital composition coefficients, real terms (with elements $n_{ij}^{base}$) |
| $n_{kj}^{baseSect}$ | Sectoral capital composition coefficients: contribution of sector $i$ to a unit of productive capacity in country-sector $j$, real terms (elements of matrix $\mathbf{N}^{baseSect}$) |
| $\mathbf{N}^{baseSect}$ | Matrix of sectoral capital composition coefficients, real terms (with elements $n_{kj}^{baseSect}$) |
| $\gamma_{ij}$ | Armington share parameters for investment goods ($\sum_z \gamma_{zj} = 1$) |
| $\theta_{kj}$ | Armington price elasticity for investment goods delivered by country-sector $k$ to country-sector $j$ |
| $h_i$ | Propensity to invest: fraction of gross output invested by country-sector $i$, real terms (elements of diagonal matrix $\hat{\mathbf{h}}$) |
| $\hat{\mathbf{h}}$ | Diagonal matrix with propensity to invest on the main diagonal, real terms (diagonal elements $h_i$) |
| $\mathbf{N}$ | Investment flow matrix, with deliveries of investment goods by the row country-sector to the column country-sector, real terms |
| $\mathbf{Q}$ | Column vector of gross output, real terms |
| $\hat{\mathbf{Q}}$ | Diagonal matrix with gross output of the country-sector on the main diagonal, real terms |
| $\mathbf{v_0}$ | Column vector of zeros |
| $\delta_i$ | Capital depreciation rate in country-sector $i$ |
| $\kappa^{pc}$ | The capital stock in productive capacity units in a country-sector, real terms |
| $I_i^{pc}$ | Investment in terms of productive capacity units in country-sector $i$, real terms |
| $\bar{u}_i$ | Neutral capacity utilization rate in country-sector $i$ |
| $\xi$ | Adjustment speed parameter in investment accelerator equation |
| $\kappa_i^{val}$ | Capital stock in historical prices in country-sector $i$, nominal terms |
| $\varpi$ | Adjustment speed parameter in dynamic markup equation |
| $\bar{d}_i$ | Neutral value for the debt-to-capital ratio (leverage ratio) in country-sector $i$ |
| $\zeta_i$ | Propensity to spend on consumption out of financial wealth stock by households (workers) in country $i$ (elements of diagonal matrix $\hat{\boldsymbol{\zeta}}$) |
| $\hat{\boldsymbol{\zeta}}$ | Diagonal matrix with $\zeta_i$ on the main diagonal |
| $\hat{\boldsymbol{\Omega}}$ | Diagonal matrix of financial wealth stocks of households, per country, nominal terms |
| $\hat{\mathbf{Z}}$ | Diagonal matrix with total nominal autonomous consumption spending per country on the main diagonal |
| $\bar{E}_i$ | Neutral employment rate in country $i$ |
| $E_i$ | Employment rate in country $i$ |
| $\iota$ | Adjustment speed parameter for $\zeta$ |
| $\tilde{E}_i$ | Average employment rate for the past 50 periods |
| $\epsilon$ | Weight parameter for endogenous $\bar{E}_i$ |
| $\chi$ | Adjustment speed parameter for wages in response to foreign debt |
| $\Xi_i$ | Ratio of foreign debt to GDP |
| $\tilde{\Xi}_i$ | Average of ratio of foreign debt to GDP over a past period |
| *Specific notation used for the transactions matrix and balance sheets* | |
| $C_k^i$ | Value of consumption goods produced in country $i$ and sold to country $k$ |
| $U_k^i$ | Value of intermediate goods produced in country $i$ and sold to country $k$ |
| $I_k^i$ | Value of investment goods produced in country $i$ and sold to country $k$ |
| $W_k$ | Total wages paid in country $k$ |

| Symbol | Explanation |
|---|---|
| $TB_k$ | Trade Balance of country $k$ |
| $O_k^i$ | Interest paid by firms in country $i$ and received by households in country $k$ |
| $D_k^i$ | Bonds issued in country $i$ and held in country $k$ |
| $\omega_i$ | Share of households in country $i$ in global financial wealth in the steady state (decision variable for steady state derivation) |
| $\lambda_i$ | Wage adjustment factor (multiplicative) for country $i$ in the steady state (decision variable for steady state derivation) |
| $inctax_i$ | Income tax variable for country $i$, only used in some implementations of the model with government; when used, $c_i$ is multiplied with $(1 - inctax_i)$ |

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**Appendix – Parameter values and initial values used in the simulations (section 5)**

This appendix describes how the simulations in Section 5 of the main text were initialized, as well as which parameter values were used in the simulations. We will use the abbreviation CxSy to denote country x – sector y.

1. The matrix of $\alpha$ values is as follows:

| | C1S1 | C1S2 | C1S3 | C2S1 | C2S2 | C2S3 |
|---|---|---|---|---|---|---|
| **C1S1** | 0.85 | 0.85 | 0.85 | 0.2 | 0.2 | 0.2 |
| **C1S2** | 0.8 | 0.8 | 0.8 | 0.15 | 0.15 | 0.15 |
| **C1S3** | 0.7 | 0.7 | 0.7 | 0.3 | 0.3 | 0.3 |
| **C2S1** | 0.15 | 0.15 | 0.15 | 0.8 | 0.8 | 0.8 |
| **C2S2** | 0.2 | 0.2 | 0.2 | 0.85 | 0.85 | 0.85 |
| **C2S3** | 0.3 | 0.3 | 0.3 | 0.7 | 0.7 | 0.7 |

2. The matrix $\mathbf{B}^{sect}$ is as follows:

| | C1S1 | C1S2 | C1S3 | C2S1 | C2S2 | C2S3 |
|---|---|---|---|---|---|---|
| **S1** | 0.1 | 0.11 | 0.09 | 0.1125 | 0.125 | 0.1375 |
| **S2** | 0.25 | 0.275 | 0.225 | 0.2025 | 0.225 | 0.2475 |
| **S3** | 0.15 | 0.165 | 0.135 | 0.135 | 0.15 | 0.165 |

3. The horizontal vector of wage rates **w** is as follows:

| C1S1 | C1S2 | C1S3 | C2S1 | C2S2 | C2S3 |
|---|---|---|---|---|---|
| 1.333333 | 0.809523 | 0.851851 | 1.916666 | 1.111110 | 0.708333 |

4. The markups **m** as a horizontal vector are

| C1S1 | C1S2 | C1S3 | C2S1 | C2S2 | C2S3 |
|---|---|---|---|---|---|
| 0.2 | 0.2 | 0.2 | 0.2 | 0.2 | 0.2 |

5. Labour coefficients **G** as a horizontal vector are as follows:

| C1S1 | C1S2 | C1S3 | C2S1 | C2S2 | C2S3 |
|---|---|---|---|---|---|
| 0.25 | 0.35 | 0.45 | 0.2 | 0.3 | 0.4 |

6. All Armington price elasticity parameters (all $\rho$s, $\varrho$s and $\theta$s) are equal to 3.8 (Bajzik et al., 2020).

7. This leads to the following horizontal price vector **p** and **B** matrix, respectively:

| C1S1 | C1S2 | C1S3 | C2S1 | C2S2 | C2S3 |
|---|---|---|---|---|---|
| 1 | 1 | 1 | 1 | 1 | 1 |

| | C1S1 | C1S2 | C1S3 | C2S1 | C2S2 | C2S3 |
|---|---|---|---|---|---|---|
| **C1S1** | 0.085 | 0.0935 | 0.0765 | 0.0225 | 0.025 | 0.0275 |
| **C1S2** | 0.2 | 0.22 | 0.18 | 0.030375 | 0.03375 | 0.037125 |
| **C1S3** | 0.105 | 0.1155 | 0.0945 | 0.0405 | 0.045 | 0.0495 |
| **C2S1** | 0.015 | 0.0165 | 0.0135 | 0.09 | 0.1 | 0.11 |
| **C2S2** | 0.05 | 0.055 | 0.045 | 0.172125 | 0.19125 | 0.210375 |
| **C2S3** | 0.045 | 0.0495 | 0.0405 | 0.0945 | 0.105 | 0.1155 |

8. All depreciation rates ($\delta$s) were set to 0.05, and all capital-output ratios ($\nu$s) were set to 3. All propensity to invest values ($h$) were initialized at 0.0556.

9. Productive capacity ($\kappa^{pc}$) and the capital stock historical prices ($\kappa^{val}$), respectively, were initialized as follows:

| C1S1 | C1S2 | C1S3 | C2S1 | C2S2 | C2S3 |
|---|---|---|---|---|---|
| 3233.865 | 5052.572 | 4156.015 | 3760.505 | 5140.195 | 4425.840 |

| C1S1 | C1S2 | C1S3 | C2S1 | C2S2 | C2S3 |
|---|---|---|---|---|---|
| 9701.594 | 15157.716 | 12468.045 | 11281.515 | 15420.585 | 13277.519 |

10. Financial wealth of households in country 1 was initialized at 22370.62, in country 2 at 24013.56. We set the interest rate to be paid on bonds to zero, and the value of outstanding bonds of firms were initialized as follows:

| C1S1 | C1S2 | C1S3 | C2S1 | C2S2 | C2S3 |
|---|---|---|---|---|---|
| -5820.96 | -9094.63 | -7480.83 | -6768.91 | -9252.35 | -7966.51 |

11. The matrix of $\beta$ values and the matrix $\mathbf{N}^{basesect}$ are, respectively, as follows:

| | C1S1 | C1S2 | C1S3 | C2S1 | C2S2 | C2S3 |
|---|---|---|---|---|---|---|
| **C1S1** | 0.9 | 0.9 | 0.9 | 0.2 | 0.2 | 0.2 |
| **C1S2** | 0.8 | 0.8 | 0.8 | 0.1 | 0.1 | 0.1 |
| **C1S3** | 0.7 | 0.7 | 0.7 | 0.3 | 0.3 | 0.3 |
| **C2S1** | 0.1 | 0.1 | 0.1 | 0.8 | 0.8 | 0.8 |
| **C2S2** | 0.2 | 0.2 | 0.2 | 0.9 | 0.9 | 0.9 |
| **C2S3** | 0.3 | 0.3 | 0.3 | 0.7 | 0.7 | 0.7 |

| | C1S1 | C1S2 | C1S3 | C2S1 | C2S2 | C2S3 |
|---|---|---|---|---|---|---|
| **S1** | 0.3 | 0.3 | 0.3 | 0.6 | 0.6 | 0.6 |
| **S2** | 1.8 | 1.8 | 1.8 | 1.5 | 1.5 | 1.5 |
| **S3** | 0.9 | 0.9 | 0.9 | 0.9 | 0.9 | 0.9 |

12. The matrix of $\gamma$ values and the matrix $\mathbf{R}^{base}$ are, respectively, as follows:

| | C1S1 | C1S2 | C1S3 | C2S1 | C2S2 | C2S3 |
|---|---|---|---|---|---|---|
| **C1S1** | 0.6 | 0.6 | 0.6 | 0.35 | 0.35 | 0.35 |
| **C1S2** | 0.7 | 0.7 | 0.7 | 0.4 | 0.4 | 0.4 |
| **C1S3** | 0.65 | 0.65 | 0.65 | 0.3 | 0.3 | 0.3 |
| **C2S1** | 0.4 | 0.4 | 0.4 | 0.65 | 0.65 | 0.65 |
| **C2S2** | 0.3 | 0.3 | 0.3 | 0.6 | 0.6 | 0.6 |
| **C2S3** | 0.35 | 0.35 | 0.35 | 0.7 | 0.7 | 0.7 |

| | C1S1 | C1S2 | C1S3 | C2S1 | C2S2 | C2S3 |
|---|---|---|---|---|---|---|
| **S1** | 0.35 | 0.35 | 0.35 | 0.45 | 0.45 | 0.45 |
| **S2** | 0.25 | 0.25 | 0.25 | 0.15 | 0.15 | 0.15 |
| **S3** | 0.4 | 0.4 | 0.4 | 0.4 | 0.4 | 0.4 |

13. All marginal propensities to consume out of wage income are equal to 0.6. No part of the operating surplus is transferred to households, i.e., the operating surplus remains in the firms.

14. The propensity to consume out of financial wealth ($\zeta$) was initialized to 0.0678 in country 1 and to 0.0644 in country 2. The labour force was set to 4002.367 in country 1 and to 3658.047 in country 2.

15. The following parameter settings were used: $\iota = 5.5, \epsilon = 0.35, \chi = 0.75, \bar{\bar{E}} = 0.93, \xi = 0.01, \varpi = 1, \bar{d} = 0.6, \bar{u} = 0.9$.